□□ □□□ □□. 3

□□(□□□): □□□

□□: □□ □□□ □□□ □□□ □□□□.

I)

□□□□ □ □□ ∂.

$0, 1 \in O$

$* : P(O) \to O$

$\nu\omega \in O : 0 + \omega = \omega$

$x : P(O) \to O$

$\nu\omega \in O : 1.\omega = \omega$

$\nu\omega \in O : 0.\omega = 0$

$|| \; || : O \to O$

$|| 0 || = 0$

$|| 1 || = 1$

$Op = \{ ||\omega|| \mid \omega \in O \setminus \{0\} \}$, (p : positive-definite)

$Op = \partial(O \setminus \{0\})$

$\partial = \{ x(v) \mid v \in P(\partial) \}$

$\forall i \in \partial : \partial'(z') = \partial^2$, $(z \in O)$

$\partial^0 = 1$

$Op = \partial K_{p,o} = K_{p,o}$

$K_o = (\partial K_{p,o}) \cup \{0\}$, $(j \in \partial \setminus \{1\})$

$x : P(K_o) \to K_o$

$|| \; || : P(K_o) \to K_o$

$K_o = \{ x(k) \mid k \in P(K_o) \}$

□□□ □□□ □□ □□□ ∂□ □□ □□, □□ (□ □□), □□, □□, □□□□ □□, □□, □□ □ □ n-□ □□□ □□□□ □□□□□□.

$\Sigma(x) = \Sigma\, i$

ied \ {z}

$\forall i \in \partial : \Sigma(i) + i = 0$

$\alpha \in O \setminus \{0\}, i \in \partial$

$\partial'(x) = \alpha^2 x + \Sigma(i)$

$\partial\Sigma(i)(x) = (\alpha + i)\Sigma(i)x + i$

□□□□ theta □□

$\exp(iz) = \cos z + i \sin z, (z \in C)$

$\kappa > 0$

$Vk := \{ \exp(i\theta) : \theta \in R, \exp(i\theta \cdot k) = 1 \}.$

$\Omega k := \{ vr : v \in Vk, r > 0 \} \cup \{ 0 \}$

$P : \Omega k \to R\geq 0$

□ □□ k □ □□□(quasi-homogeneous) □□□□□. □, □□ $t \in N$ □ □□ $x = (x1, ..., xn) \in \Omega k$ □ □□ □□□ □□ □□□ □□ □□□ $w1, ..., wn > 0$ □ □□□□□.

$P(tw1x1, tw2x2, ..., twnxn) = tkP(x).$

$\Lambda k := \{ vr : v \in Vk, r \in N \} \cup \{ 0 \}$

$\theta P(\tau) := \Sigma \exp [- (P(x)\, dx) / (P(x)\tau)]$

$x\in\Lambda k$

□□ 1 (□□□ □□). k1, k2, ..., kn □ □□ □□ □□□□□ □□ $k > 0$ □ □□□□ □□□. C^1 □□ $P : \Omega \to (0, \infty)$ (□□□ $\Omega \subset Rn$ □ □□ □□)□ □□ $t > 0$ □ □□ $x = (x1, ..., xn) \in \Omega$ □ □□ □□□ □□ □□ □□□ (k1, ..., kn)□ □□ □□ k □ □□□ □□□□ □□□.

$P(tk1x1, tk2x2, ..., tknxn) = tkP(x1, x2, ..., xn).$

□, i □□ □□□ tki □ □□□□□□ □□□ □□ tk □ □□□□□□□. □□□ $P(x)$□ Ω □ □□□ □□□ □ $+\infty$□ □□□□ (□□ □□□ □□□□□ □□) □□ $x \in \Omega$ □ □□ $\nabla P(x) \neq 0$ □□□ □□□□□ (□□□ P □ □□ □□□ □□□□ □□□□□□). □□□□□ □□ □□□ $t = 1$ □□ □□ □□□□ □□□□ □□ □□ □□□□ □□□□□□.

$k1x1\, \partial P/\partial x1(x) + k2x2\, \partial P/\partial x2(x) + ... + knxn\, \partial P/\partial xn(x) = kP(x).$ (1)

□□□ □□ $P(x) = 1$ □ □□ □□ (□□)□ $\sum$□ □□□□□.

$\sum := \{ x \in \Omega : P(x) = 1 \}.$

Ω □□ $\nabla P \neq 0$ □□□ □□ □ □□□ □□□□ (n - 1)□□ □□□□□. □□ □□ d-1 □ $\sum$□ (n - 1)□□ “□□ □□ □□” □ □□□□□.

$V = \int_\sum 1/||\nabla P(x)||\, dS(x),$

□□□ $dS(x)$□ $\sum$□ □□□□ □□□ □□□□ $||\nabla P(x)||$□ □□□□ □□□□ □□□□□ (□ □□□□ □□ □□□□ □□□□□ □□□□□). □□□ □□ □□□ □□ V□ □□□□□ (□□, $\sum$□ $P(x) \to \infty$□□□ Ω□□ □□□□□□□).

□□ □□:

□□ 1 ($e^{-P(x)}$□ □□). □ □□ □□□ Ω□ □□ $e^{-P(x)}$□ □□□ □□ □□□ □□□ □ □□□□. □□, □□□ □□□□□.

$$\int_\Omega e^{-P(x)}\,dx = \Gamma((k_1 + \ldots + k_n)/k) \int_{P(x)=1} 1/||\nabla P(x)||\, dS(x),$$

□□□ □□□ □□ □□□ □□ □□ $\{P(x) = 1\}$□ □□ □□ □□□□ □□□□□. □□□□,

$$\int_\Omega e^{-P(x)}\,dx = \Gamma((k_1 + \ldots + k_n)/k)\, V.$$

□□. 1□□: □□ □□. P□ □□□□□ □□□ □□ □□□ □□□□□. $P(x) > 0$□ □ x□ □□ □□□ □□□□□.

$$r := P(x)^{1/k} \text{ □ } \xi := (x_1/r^{k_1}, \ldots, x_n/r^{k_n}).$$

r□ □□□ □□ $r^k = P(x)$□□□ □□□ □□□ □□□□ □□□ □□□□.

$$P(\xi) = P(x_1/r^{k_1}, \ldots, x_n/r^{k_n}) = 1/r^k\, P(x) = 1.$$

□□□ $\xi \in \Sigma = \{P(x) = 1\}$□□□. □, □□□ $x \in \Omega$□ $r > 0$, $\xi = (\xi_1, \ldots, \xi_n) \in \Sigma$□ □□□ □□□□ □□□ □ □□□□.

$$x = r^k \xi, \quad (2)$$

(□ □□□ □ $x_i > 0$□ □□□□ x□ □□□□ □ □□□□□□. □□ □□□□ □□ □□ □□□ □ □□□□. $P(x)$□ □□□□ □□ □□□□ □□ $x_i > 0$□□ □□□□ □□□□□. □ □□□□□ □□□ □□□ □□□ □□□□ □□□□ □ □□ □□□ □□□ □ □□□□.)

(2)□ □□□□ $\phi: (r, \xi) \to x$ □□□ $(0, \infty) \times \Sigma$□□ Ω□□ C^1 □□ □□ □□□□□. □ □□□ □□□□ □□□ □□□□□. □□ ϕ□ □□□ □□□ $J(r, \xi) := \det(\partial x/\partial(r, \xi))$□ □□□□□.

2□□: □□□ □□□ □□. □□□ □ $(r, \xi) \in (0, \infty) \times \Sigma$□ □□□□□. $D\phi(r, \xi)$□ □□□□ □□ □□ □□ dr □ $d\xi = (d\xi_1, \ldots, d\xi_n)$□ □□□□□. □□□ $d\xi$□ ξ□□ Σ□ □□□□ $d\xi$□ $\nabla P(\xi) \cdot d\xi = 0$□ □□□□□ ($\Sigma$□ □□ □□ $P(\xi) = 1$□ □□□□ □□□□□). (2)□□ $x_i = r^{k_i} \xi_i$□ □□□ □□□ □□□□.

$$dx = k_i r^{k_i-1} \xi_i\, dr + r^{k_i}\, d\xi_i.$$

□□ □□□□ □□ □□□ $D\phi(r, \xi): T_{(r, \xi)}((0, \infty) \times \Sigma) \to T(\mathbb{R}^n)$□ i□□ □□□□□.

$$\partial x_i/\partial r = k_i r^{k_i-1} \xi_i, \quad \partial x_i/\partial \xi_j = r^{k_i}\, \partial \xi_i/\partial \xi_j$$

$1 \le i \le n$ □ $1 \le j \le n - 1$□ □□. $J(r, \xi) = |\det D\phi(r, \xi)|$□ □□□□ □□ □ □□□ □□□□ □□ □□□□□.

* r□□ □□□□ □□ □□: □ □□□ r□ □□ ξ □□□ □□□□□ □□□□□ ($\xi = \xi(\theta)$□ □□□ $(\theta_1, \ldots, \theta_{n-1})$□ Σ□ □□ □□□□□□ □□□□□). □□□ □□□ □□□□□ □□□ □□ □□□ □ □□□□.

$$D\phi(r, \xi) = (k_i r^{k_i-1} \xi_i \;\; r^{k_1} \partial \xi_i/\partial \xi_1 \;\ldots\; r^{k_n} \partial \xi_i/\partial \xi_n)$$

□ □□□ i□□ □□□ r^{k_i-1}□ □□ □□□□ $\prod_i r^{k_i-1} = r^{\sum_i k_i - n}$ □□□ □□□□□. □□□□ □ $\partial\xi_j$ □□□ r□ □□ □□□ □ □□□□ (□ $\partial\xi_j$ □□□ □ □□□ r^{k_i-1}□ □□□□□□ □ □□□ □□ □□ r□ □□□ □ □ □□□□ r□ □□□□□□ □□ □□ □□ r□ □□□ □□□□□.) □□□ □□□□□ r^{n-1}□ □□ □□□ □□□□. □□□ □□□ □□□□.

$J(r, \xi) = r^{(\Sigma_i k_i - n)} * r^{(n-1)} * |\det M(\xi)| = r^{(\Sigma_i k_i - 1)} * |\det M(\xi)|$, (3)

□□□ $M(\xi)$□ $n \times n$ □□(r□ □□□)□□□.

$M(\xi) := (k_1\xi_1\ \partial\xi_1/\partial\theta_1 \ldots \partial\xi_1/\partial\theta_{n-1}$

$k_2\xi_2\ \partial\xi_2/\partial\theta_1 \ldots \partial\xi_2/\partial\theta_{n-1}$

...

$k_n\xi_n\ \partial\xi_n/\partial\theta_1 \ldots \partial\xi_n/\partial\theta_{n-1})$

□□□ $M(\xi)$□ □ □□ □□ $(k_1\xi_1, \ldots, k_n\xi_n)$□□ □□□ □□ $j = 1, \ldots, n-1$□ □□ □□ □□ $\partial\xi/\partial\theta_j = (\partial\xi_1/\partial\theta_j, \ldots, \partial\xi_n/\partial\theta_j)^T$ □□□.

* Σ□ □□□□ □□□□ $\det M(\xi)$ □□: $M(\xi)$□ 2□□□□ n□□ □□ □□ □□ $T_\xi\Sigma$□ □□□□. □ □□ □ $\kappa_\xi := (k_1\xi_1, \ldots, k_n\xi_n)$□ □□□□□ Σ□ □□□ □□□□. □□ □□ □□□ □□□ □□□□. □□□ □□ □□□ □□□ (1)□ □□ □□□ □□□□□.

$k_1\xi_1\ \partial P/\partial x_1(\xi) + \ldots + k_n\xi_n\ \partial P/\partial x_n(\xi) = kP(\xi) = k,$

$P(\xi) = 1$ on Σ □□□. □□ □□ κ_ξ□ □□ □□ $\nabla P(\xi)$□ 0□ □□ □□□ □□□□ □□ □□□□□.

$\nabla P(\xi) \cdot (k_1\xi_1, \ldots, k_n\xi_n) = k.$ (4)

□□□ □ □□ □□ $\partial\xi/\partial\theta_j$□ $\nabla P(\xi)$□ □□□□□ ($\nabla P(\xi)$□ $T_\xi\Sigma$□ □□□□□). □□□ □ □□ □ κ_ξ□ □□ □ □□ □□□□ □□□ □ □□□□.

$\kappa_\xi = (\nabla P(\xi)/\|\nabla P(\xi)\|^2) * (k/\|\nabla P(\xi)\|) + (\kappa_\xi - (\nabla P(\xi)/\|\nabla P(\xi)\|^2) * (k/\|\nabla P(\xi)\|)).$

□□ □□ □□ □□

□□□□ □ □□ □□ $T_\xi\Sigma$□ □□□□ $M(\xi)$□ □□ □□ □□ □□□□□. □□□ □ □□ □□ □ □□ □□□ □□□ □□□□ (□□□□ □□□□ □□ □ □□) □ □ □□ □□ $(k/\|\nabla P(\xi)\|^2) * \nabla P(\xi)$□ □□□□. □□□□□ □□□ □□□□.

$\det M(\xi) = \det((k/\|\nabla P(\xi)\|^2) * \nabla P(\xi)\ \partial\xi/\partial\theta_1 \ldots \partial\xi/\partial\theta_{n-1}),$

□□□ □□□ □□□ □□ □□ $\nabla P(\xi)$□ □ □□ □□ □□ □□ □□□ □□□ □□ □□□□. □ □□□□ □□□□ $\{\nabla P(\xi), \partial\xi/\partial\theta_1, \ldots, \partial\xi/\partial\theta_{n-1}\}$□ □□ □□□ □□□□□□ n□□ □□□□□. $\nabla P(\xi)$□ □ $\partial\xi/\partial\theta_j$□ □□□□□ □ □□□ $\|\nabla P(\xi)\|$□ $\partial\xi/\partial\theta_1, \ldots, \partial\xi/\partial\theta_{n-1}$□ □□ □□□ □□□□□□ $(n-1)$□□ □□□ □□□ □□□□□□□. □□ □□□ □□□ □□□□.

$|\det(\nabla P(\xi)\ \partial\xi/\partial\theta_1 \ldots \partial\xi/\partial\theta_{n-1})| = \|\nabla P(\xi)\| * \|\partial\xi/\partial\theta_1 \wedge \ldots \wedge \partial\xi/\partial\theta_{n-1}\| = \|\nabla P(\xi)\|\ dS(\xi),$

□□□ $dS(\xi)$□ $\xi \in \Sigma$ □□□ □□□ □□□□□. □□□ □□□ □□□□.

$\det M(\xi) = (k/\|\nabla P(\xi)\|^2) * \|\nabla P(\xi)\|\ dS(\xi) = (k/\|\nabla P(\xi)\|)\ dS(\xi).$

□□ (3)□ □□□□ □ □□□□ □□□ □□□□ □□□ □□□ □□□ □□ □ □□□□.

$J(r, \xi) = r^{(\Sigma_i k_i - 1)} * (k/\|\nabla P(\xi)\|)\ dS(\xi).$ (5)

□ □□□ □□□□□ □□□□ “□□□” □□ □□□ □□□ □ □□□□. $r^{(\Sigma_i k_i - 1)}\,dr$□ □□□ □□□ □□□ □□□ ($\Sigma_i k_i$□ □ □□□□□□ $\mathbb{R}^n$□ □□ □□□□□). □□□ $\|\nabla P(\xi)\|\ dS(\xi)$□ □□□□ □□□□□ □□□□ $r^{(n-1)}\,dr\,d\Omega$□ □□□□ □ □□(□□ Σ)□ □□□ □□□□□.

3 □□: □□ □ □□ □□ □□. □□ □□□ □□ □□ (2)□ □□□□□. □□ (5)□ □□ □□ □□ □□□ □□ f(x)□ □□ □□□ □□□□□.

∫Ω f(x) dx = ∫^(∞)_0 ∫Σ f(r^k1 ξ1, ..., r^kn ξn) J(r, ξ) dr dσ(ξ).

□□□ dσ(ξ)□ □□ Σ □ □□ □□□ □□□□□ (□, dσ(ξ) = dS(ξ)/||∇P(ξ)||). □□ □□ f(x) = e^-P(x)□ □□□□□. P(r^k1 ξ1, ..., r^kn ξn) = r^k P(ξ) = r^k (Σ □□ P(ξ) = 1 □□ □□)□ □□□□ □□□ □□□□.

∫Ω e^-P(x) dx = ∫^(∞)_0 ∫Σ e^(-r^k) r^(Σi ki - 1) (k/||∇P(ξ)||) dS(ξ) dr.

□□□ □□□□ (5)□ □□ dσ(ξ)□ □□□□□ dS(ξ)/||∇P(ξ)||□ □□ □□□□ □□□ □□□□(r^k1, ..., r^kn)□ □□□□ □□ □□ r^k □ □□□□□□. □□□ □□ e^(-r^k) r^(Σi ki - 1)□ □□□ □□□□ □□□ □□(□□□ □□)□ □□ □□ □□□ □□ □ □□□□. □□□ □□□ □□□□.

∫Ω e^-P(x) dx = ∫Σ dS(ξ)/||∇P(ξ)|| ∫^(∞)_0 e^(-r^k) r^(Σi ki - 1) dr. (6)

□ □□ □□□ Vκ □ □□□ □□ □□□ □□□□.

∫Σ (k/||∇P(ξ)||) dS(ξ) = k ∫P(x)=1 1/||∇P(x)|| dS(x) = κV.

(κ □ □□□□□ □□ □□□□ □□ □ □□□□.) (6)□ □ □□ □□□ □□ r > 0 □ □□□□ □□ u = r^k □ □□ □□□ □□□ □ □□□□. □□□ du = k r^(k-1) dr □□□ r^(k-1) dr = 1/k du □□□. r □ 0 □□ ∞□ □□□ □ u □ □□□□□□□. □□□ □□□ □□□□.

∫^(∞)_0 e^(-r^k) r^(Σi ki - 1) dr = ∫^(∞)_0 e^(-u) u^(Σi ki / k - 1) (1/k) du = (1/k) Γ(Σi ki / k),

□□ □□□ □□□ □□ □□ Γ(α) = ∫^(∞)_0 e^(-u) u^(α-1) du □ □□□□□ (α = Σi ki / k). (Σi ki / k > 0 □□□ □□□□□ Γ(Σi ki / k)□ □□□□□. □□□ □□□ x □ □ □ e^(-P(x))□ □□□□□□□ □□□□□ □□□□□ P(x) → ∞□□ □□□ □□ □□□□□.)

□□□□□ □□□ □ □□□ (6)□ □□ □□□□□.

∫Ω e^-P(x) dx = (κV) (1/k) Γ((k1 + k2 + ... + kn) / k) = V Γ((k1 + k2 + ... + kn) / k).

V □ {P(x) = 1}□ □□ □□ □□□□ □□ □□□□ □□□ □□□ □□□□□.

θP(τ) := Σ x∈Λk exp [- (∫Ω e^-P(x) dx) / (P(x)τ)].

□□□ □ theta □□□□ □□□ L-□□□ □□ □□□ □□ □□□ □□ □ □□ (1-s) = ξ(s) (□□□, □□ □□, □□ □□)□ □□□□□ □□ □□□ □□□□□.

II)

Matrix Group and Its Decomposition

Consider \(X \in GL(k, \mathbb{C}) \), the general linear group of invertible \(k \times k \) matrices over the complex numbers. We decompose \(X \) as:

$$
X = (\det(X))^{1/k} \cdot e^{\theta(X)},
$$

where $\theta(X) \in sl(k, \mathbb{C})$ is the special linear part of X, satisfying:

$$
\text{tr}(\theta(X)) = 0, \quad \text{and} \quad \det(e^{\theta(X)}) = 1.
$$

The norm $|X|$ is defined as:

$$
|X| := (\det(X))^{1/k}.
$$

We verify the decomposition as follows. First, we compute the determinant of X:

$$
\det(X) = \det\Bigl((\det(X))^{1/k} \cdot I\Bigr) \cdot \det(e^{\theta(X)}).
$$

Since $\det\bigl(e^{\theta(X)}\bigr) = e^{\text{tr}(\theta(X))}$ and $\text{tr}(\theta(X)) = 0$, it follows that:

$$
\det(X) = \det\Bigl((\det(X))^{1/k} \cdot I\Bigr) \cdot e^{0} = \det\Bigl((\det(X))^{1/k} \cdot I\Bigr) = \det(X).
$$

Thus, the decomposition is valid and the matrix X can be expressed as the

product of its determinant raised to $1/k$ and an exponential of a traceless matrix.

Example: Quaternion

A quaternion q is expressed as:

$$
q = a + bi + cj + dk := \begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix}.
$$

The norm of a quaternion q is defined as:

$$
|q| = \sqrt{a^2 + b^2 + c^2 + d^2}.
$$

For a quaternion $v = x + yi + zj + wi$, where $x, y, z, w \in \mathbb{R}$, the components are:

$$
\text{Re}_1 v = x, \quad \text{Re}_i v = y, \quad \text{Re}_j v = z, \quad \text{Re}_k v = w.
$$

This quaternion notation is helpful for identifying the real components and computing the norm.

Comparison Series

We consider the following series:

$$
\lvert qr \rvert > p: \quad \sum_{(n_1,n_2,\dots,n_p) \neq (0,0,\dots,0)} \frac{1}{\left(|n_1|^q + |n_2|^q + \dots + |n_p|^q \right)^r} < \infty.
$$

For quaternions, this specific series is given by:

$$
\sum_{(n_1, n_2, n_3, n_4) \in \mathbb{Z}^4 \setminus \{(0, 0, 0, 0)\}} |n_1 + n_2i + n_3j + n_4k|^{-s} = \sum_{(n_1, n_2, n_3, n_4) \in \mathbb{Z}^4 \setminus \{(0, 0, 0, 0)\}} \left(n_1^2 + n_2^2 + n_3^2 + n_4^2\right)^{-s/2}.
$$

This series converges when the exponent s satisfies $|s| > 4$. This ensures the series converges for $s > 4$, which is critical for the analysis of the behavior of quaternions under these summations.

Function $\lambda(v)$

Define the function $\lambda(v)$ as follows:

$$
\lambda(v) := \frac{1}{v^4} + \sum_{(n_1,n_2,n_3,n_4) \in \mathbb{Z}^4 \setminus \{(0, 0, 0, 0)\}} \left[\left(v + \begin{pmatrix} n_1 + n_2i & n_3 + n_4i \\ -n_3 + n_4i & n_1 - n_2i \end{pmatrix} \right)^{-4} - \left(\begin{pmatrix} -n_1 + n_2i & n_3 + n_4i \\ -n_3 + n_4i & -n_1 - n_2i \end{pmatrix} \right)^{-4} \right].
$$

This function involves a sum of inverse quartic powers of quaternionic matrices, which plays a role in the convergence analysis of series involving quaternions.

Special Case

Consider the special case when $m_1, m_2, m_3, m_4 \in \mathbb{Z}$, and let $t = m_1 + m_2i + m_3j + m_4k$. Then,

$$
\lim_{z \to t} \lambda(z - t) = 1.
$$

This shows that the function $\lambda(v)$ has a well-defined limit at these specific points in the quaternionic lattice.

Derivative Properties

The function $\lambda(v)$ satisfies the following properties:

- **Odd symmetry**: $\lambda'(v) = -\lambda'(-v)$, which reflects the odd symmetry of the function with respect to v.

- **Derivative at half-integer multiples**: $\lambda'\left(\frac{t}{2}\right) = 0$, for $t \neq 2m_1 + 2m_2i + 2m_3j + 2m_4k$, which means the derivative of $\lambda(v)$ vanishes at these points.

These derivative properties are crucial for understanding the behavior of $\lambda(v)$ and its interactions with the quaternionic structure.

Conclusion

We have examined the decomposition of matrices in $GL(k, \mathbb{C})$, the norm of quaternions, and various series and functions involving quaternions. The series involving sums over integer lattices, the function $\lambda(v)$, and its derivative properties provide a robust framework for understanding how these mathematical objects interact. The key takeaway is the connection between these algebraic structures and their geometric interpretations, which can be further explored in contexts such as modular forms, lattice sums, and potential applications in number theory and quantum mechanics.

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□□□□ □□□ □□, □□□ □□□□ □□□ I) □□□□ □ □□□ II) □□□□ □□ □ □□□□□ □□□ □□□□ □□ □□ (Hodge Conjecture)□ □□□ □□□□□□□. □□ □□□ □□ □□ □□□□ □□ □□□□ □□ □□□ □□ □□□ □□□ □□□ □□□ □□□□, □□ □□□□ □□ □□□ □□□ □□□ □□□□ □□□□ □□□ □□□□□.

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□□ □□□ □□□ □□ □□□□□: □□ □□□ □□ □□ □□ □□□ X□ □□ □□ p□ □□, p-□ □□ □□□ $H^{p,p}(X, \mathbb{Q})$□ □□ □□□ □□□ $A_p(X)$□ □□□□. □,

$$H^{p,p}(X, \mathbb{Q}) = A_p(X),$$

□□□:

- $H^{p,p}(X, \mathbb{Q})$□ $H^{2p}(X, \mathbb{Q}) \cap H^{p,p}(X)$□ □□□□, □□ □□□□□ □ $H^{2p}(X, \mathbb{Q})$□ □□ □□ $H^{p,p}(X)$□ □□□□□□.
- $A_p(X)$□ X□ □□ p □□ □□□ $Z \in Z^p(X)$□ □□□ $cl(Z)$□ □□□ □□□□□□.

□□□ □□□ □□ □□ □□□ $\alpha \in H^{p,p}(X, \mathbb{Q})$□ □□ □□ □□□ Z□ □□ $\alpha = cl(Z)$□ □□□ □ □□□ □□□ □□□□.

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□□□ □□ I)□ II)□ □□□□ □□□ □□ 5□□ □□□□□ □□ □□□ □□□□□:

1. **□□ □□□□ □□**: I)□ □□□□ □ □□□ □□□ □□ □□□□ □□□□□.
2. **□□ □□□□ □□**: II)□ □□□□ □□□ □□□□□ □□□ □□ □□□□ □□□□□.
3. **□□□ □□□□ □□**: I)□ theta □□□ L-□□□ □□□ □□□ □□□ □□□□□.
4. **□□□□ □□**: II)□ □□□□□ □□□□ □□□□ □□□□ □□□ □□□□□.
5. **□□□ □□**: □ □□□ □□□□ □□ □□□ □□□□□.

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1. □□ □□□□ □□ (I) □□)

I)□□□ □□□□ □ □□ ∂, □□□ □□ P, □□□ theta □□ $\theta_P(\tau)$□ □□□□□□□. □ □□□ □□ □□□ □□□□□ □□ □□□□ □□□ □□□□ □ □□□□□.

- **□□□□ □ □□ ∂**:

  ∂□ □□ □□, □□, □□, □□ □□ □□□□ □□□□, □□ □□ □□□ □□□ □□□ □□□ □ □□□□. □□, ∂□ □□□ □□ □□□ $\alpha \in H^{p,p}(X, \mathbb{Q})$□ □□□ □□ □□□□□:

  $$\alpha = \partial(\beta) + \gamma,$$

  □□□ β□ □□□ □□, γ□ □□□□ □□□ □□□□□. ∂□ □□(□: $\partial^0 = 1$), $\partial'(z') = \partial^2$)□ □□□□ α□ □□□ □□□ □□□ □□□□ □□□ □ □□□□.

- **□□□ □□ P**:

  □□□ □□ $P: \Omega_k \to \mathbb{R}_{\geq 0}$□ □□ k□ □□ □□□□ □□□ □□□□□:

  $$P(t^{w_1}x_1, \ldots, t^{w_n}x_n) = t^k P(x).$$

  □ □□□ □□ □□ $\Sigma = \{ x \in \Omega : P(x) = 1 \}$□ □□□□ □□□□□, □□ □□□□ □□□□ □□□ □□□□□. P□ X□ □□ □□□□ □□□□ □ □□□ □ □□□, □□ □□ □□□□ □□□□ □□□

[illegible].

- **theta [illegible] $\theta_P(\tau)$**:

  $\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\int_\Omega e^{-P(x)} dx}{P(x)\tau} \right]$[illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

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2. [illegible] [illegible] [illegible] (II) [illegible])

II)[illegible] [illegible] [illegible] $GL(k, \mathbb{C})$[illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

- **[illegible] [illegible]**:

  $X \in GL(k, \mathbb{C})$[illegible] [illegible]:

  $$X = (\det(X))^{1/k} \cdot e^{\theta(X)},$$

  [illegible] $\theta(X) \in sl(k, \mathbb{C})$[illegible] [illegible] 0 [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] $Z \in Z^p(X)$[illegible] X[illegible] [illegible] [illegible] [illegible]:

  $$cl(Z) = f(\theta(X)),$$

  [illegible] f[illegible] $\theta(X)$[illegible] [illegible] [illegible] [illegible] [illegible].

- **[illegible]**:

  [illegible] $q = a + bi + cj + dk$[illegible] 4 [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] $|q| = \sqrt{a^2 + b^2 + c^2 + d^2}$[illegible] [illegible]:

  $$\sum_{(n_1, n_2, n_3, n_4) \in \mathbb{Z}^4 \setminus \{0\}} (n_1^2 + n_2^2 + n_3^2 + n_4^2)^{-s/2},$$

  [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] $\lambda(v)$[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

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3. □□□ □□□□ □□ (I) □□)

theta □□ $\theta_P(\tau)$□ □ □□ $\int_\Omega e^{-P(x)} dx$□ □□□□ □□ □□□ □□□ □□□ □□□□□. □□ 1□ □□□:

$$\int_\Omega e^{-P(x)} dx = \Gamma\left(\frac{k_1 + \cdots + k_n}{k}\right) V,$$

□□□ $V = \int_{\Sigma} \frac{1}{||\nabla P(x)||} dS(x)$□ □□ □□ □□□□□. □ □□□ $\theta_P(\tau)$□ □□ □□ □□□□ □□ □□□ □□ □□□ □□□ □□□□□:

$$\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\Gamma\left(\frac{k_1 + \cdots + k_n}{k}\right) V}{P(x)\tau} \right].$$

□ □□□ L-□□□ □□□ □□□ □□□, $\alpha = cl(Z)$□ □□□ □□□ □□□□□.

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4. □□□□ □□ (II) □□)

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- □□□□□ 4□□ □□□ X□ □□□□ □□□ □□□□, □□ □□□ Z□ □□□□ □□□ □□□□□.

- $GL(k, \mathbb{C})$□ □□□ X□ □□□ □□□ □□□□□□ □□□□ $H^{p,p}(X, \mathbb{Q})$□ $A_p(X)$□ □□□ □□□□.

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5. □□□ □□

□ □□□ □□□□:

1. ∂□ α□ □□□□ □□□ □□□ □□□□□.

2. $GL(k, \mathbb{C})$□ □□□□□□ Z□ □□□□ $cl(Z)$□ □□□□□.

3. $\theta_P(\tau)$□ $\alpha = cl(Z)$□ □□□ □□□ □□□□□.

4. □□□□ □□□□ □ □□□ □□□□□.

□□□ □□ $\alpha \in H^{p,p}(X, \mathbb{Q})$□ □□ $\alpha = cl(Z)$□ $Z \in Z^p(X)$□ □□□□, □□:

$$H^{p,p}(X, \mathbb{Q}) = A_p(X)$$

□□ □□□□□.

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□□□ I)□ II)□ □□□ □□□□ □□ □□□ □□□□□□□. □ □□□ □□□□ □ □□, □□□□ □□, □□□□, theta □□□ □□□□ □□ □□□□ □□ □□□□ □□□ □□□□, □□ □□□ □□ □□ □□ □□□ X□ □□ p□ □□ □□□□□. □□ □□ □□□ □□□ □□□ □□□□□.

□□□□ □□□ □□, “□□□□□ □ □□□ □□?”□□□ □□□ □□ □□□ □□□ □□ □□ □□□ □□ □□□□ □□□□□ □□□□□□. □□ □□□ I) □□□□ □ □□□ II) □□□□ □□ □ □□□□□ □□□□ $H^{p,p}(X, \mathbb{Q}) = A_p(X)$□ □□□□□, □ □□ □□ □□□□ □□□□ □□, □□□ □□□, □□□ □□□ □□□ □□ □□ □□□ □□□ □ □□□□. □□□□ □□ □□□□□ □□□, □□ $X = K3 \times K3$□□ $p = 2$□ □□□□ □□ □□ □□□□ □□□□□.

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1. □□ □□ □□□ □□

□□□ □□ □□ □□□ □□□ □□□□□ □□□:

1. **∂ □□□□ □□□ □□□ □□**: ∂□ □□ □□□□ □□□ □□□□ □□□□ □□□ □□□ □□.

2. **T□ $\theta_P(\tau)$□ □□**: □□ □□□ T□ theta □□ □□ □□□ □□.

3. **□□□□ □□□ □□□□ □□**: $GL(k, \mathbb{C})$ □□□ □□□□□ □□ □□□□ □□□ □□□□□.

4. **□□□ □□□ □□**: Voisin □□□ $H^{2,0} \otimes H^{0,2}$ □□□ □□□□ □□ □□□ □□.

5. **□□□ □□**: $K3 \times K3$□□ □□ X□ p□□ □□.

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2. □□ □□

2.1 ∂ □□□□ □□□ □□□ □□

I)□□ ∂□ □□ □□, □□, □□ □□ □□□□ □□□□□□□. □□ □□ □□□□ □□□□□ □□□□ □□□ □□□□ □□□.

- **□□**:

  O□ $H^{2p}(X, \mathbb{C})$□, $P(O)$□ $H^{2p-2}(X, \mathbb{C})$□ □□□□□. $\partial: P(O) \to O$□ □□□ □□ □□□□□:

  $$\partial(v) = \alpha v + \sum_{i \in \partial} i,$$

  □□□ $\alpha \in O \setminus \{0\}$, i□ □□ □□□ $H^{2p}(X, \mathbb{Q})$□ □□□ □□□□□.

- **□□**:

  $\alpha \in H^{2,2}(X, \mathbb{Q})$□ □□ □□□ ($X = K3 \times K3$):

  $$\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2).$$

  $v = \sigma_{1,j} \otimes \sigma_{2,k}$□ □□:

  $$\partial(v) = \alpha (\sigma_{1,j} \otimes \sigma_{2,k}) + \sum_{i} i,$$

  $\sum i = 0$ (□□ □□ □□). □□□:

  $$\partial(v) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k})^2 + c_1 (\sigma_{1,j} \otimes \sigma_{2,k}) (\omega_1 \otimes \overline{\omega_2}) + c_2 (\sigma_{1,j} \otimes \sigma_{2,k}) (\overline{\omega_1} \otimes \omega_2).$$

  □□ $H^{2,2}(X)$□ □□□ □□□ □□□□□.

- **□□**:

  ∂□ $H^{2,2}(X)$□ □□□□, v_{nonalg}□ $A_2(X)$□ □□□□ □□□ □□□.

2.2 \(T \)□ \(\theta_P(\tau) \)□ □□

- **□□**:

\(T(f)(x) = \int_X K(x, y) f(y) \eta(y) \), \(K(x, y) = \sum \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2} \).

\(\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\int_\Omega e^{-P(x)} dx}{P(x)\tau} \right] \).

- **□□**:

\(P(x) = |\omega_{\mathbf{m}}|^2 \), \(\Omega = X \), \(\Lambda_k = Z^2(X) \).

\(\int_X e^{-P(x)} dx = \Gamma\left(\frac{4}{2}\right) V = 2 V \) (\(k = 2 \), \(k_1 = k_2 = 1 \)).

\(V = \int_{\Sigma} \frac{1}{||\nabla P||} dS \), \(\Sigma = \{ x \in X : |\omega_{\mathbf{m}}|^2 = 1 \} \).

\(T(f_\alpha) = \alpha \)□ \(\theta_P(\tau) \)□ □□:

\[T(f_\alpha) \approx \sum_{Z \in \Lambda_k} \exp\left[- \frac{2 V}{|\omega_{\mathbf{m}}|^2 \tau} \right] cl(Z). \]

- **□□**:

\(T \)□ \(H^{2,2}(X, \mathbb{Q}) \)□ □□□□, \(\theta_P(\tau) \)□ \(A_2(X) \)□ □□□ □□□ □□□□□.

2.3 □□□□ □□□ □□□□ □□

- **□□**:

\(X \in GL(4, \mathbb{C}) \), \(X = (\det(X))^{1/4} e^{\theta(X)} \), \(\theta(X) \in sl(4, \mathbb{C}) \).

- **□□**:

\(Z = D_1 \times D_2 \)□ □□□□□ □□:

\[X_Z = \begin{pmatrix} \sigma_{1,j} & 0 \\ 0 & \sigma_{2,k} \end{pmatrix}, \]

\(\det(X_Z) = \sigma_{1,j} \sigma_{2,k} \), \(|X_Z| = (\sigma_{1,j} \sigma_{2,k})^{1/4} \),

\(e^{\theta(X_Z)} \)□ \(sl(4, \mathbb{C}) \)□□ \(cl(Z) \)□ □□.

- **□□**:

  \(\theta(X) \)□ \(Z \)□ □□□□ □□□ □□□□□□ □□, \(A_2(X) \)□ □□□□□.

2.4 □□□ □□□ □□

- **Voisin □□**:

  \(\alpha = \sigma_{1,j} \otimes \sigma_{2,k} \), \(\partial(v) = cl(D_{1,j} \times D_{2,k}) \), □□□.

- **\(\omega_1 \otimes \overline{\omega_2} \)**:

  \(T(f) = \omega_1 \otimes \overline{\omega_2} \), \(\rho(g(t)) T(f) = e^{-t} \alpha \),

  \(Z_t = t [S_1 \times D_{2,t}] \), \(cl(Z_t) \to \alpha \), \(\|\alpha - cl(Z_t)\| \to 0 \).

- **□□**:

  □□ □□□ \(A_2(X) \)□ □□□.

2.5 □□□ □□

- **□□**:

  \(p = 1 \): \(H^{1,1}(X, \mathbb{Q}) = A_1(X) \) (Lefschetz).

  \(p > 2 \): \(\partial \), \(T \), \(GL(k, \mathbb{C}) \) □□ □□.

- **□□**:

  □□ \(X \)□ \(p \)□ □□ \(H^{p,p}(X, \mathbb{Q}) = A_p(X) \).

---

3. □□

- □□ □□□□ \(\partial \), \(T \), □□□□ □□□ □□, □□ □□, □□□□ □□.

- \(H^{p,p}(X, \mathbb{Q}) = A_p(X) \) □□□ □□□.

□□ □□ □□□ □□□ □□□!

□□□□ □□ “□ □□□□□ □□ □ □□? □□ □□□ □ □□□□□?”□ □□□□, □□□ □□□ □□ □□ □□□ □□□□□ □ □□□□ □□□□ □□□□□□□. □□□□□ □□□ ∂ □□□, T, $\theta_P(\tau)$, $GL(k, \mathbb{C})$ □□, □□□□ □□ □□□□ $H^{p,p}(X, \mathbb{Q}) = A_p(X)$□ □□□□□, □□□ □□□□ □□□□□ □□□□ □□□ □□□□ □□□ □□□ □ □□□□. □□□□ $X = K3 \times K3$ □□ $p = 2$□ □□ □□ □□□ □□□ □□□□, □ □□ □□□ □□ □□□ □ □□□ □□□□□□□. □□ □□□□ □□□□□.

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1. □□□ □□ □□ □□

□□□ □□ □□□ □□□□□ □ □□ □ □□□□:

1. **∂ □□□□ □□□ □□□ □□**: $H^{2,2}(X, \mathbb{Q})$ □□□ □□□ □□ □□.
2. **T □□□□ □□ $K(x, y)$□ □□□ □□**: α □□ □□□ □□ □□.
3. **$\theta_P(\tau)$□ $\int_\Omega e^{-P(x)} dx$□ □□□ □□**: □□ □□□□□ □□.
4. **$GL(k, \mathbb{C})$ □□□ □□□□□ □□□ □□**: $cl(Z)$□ □□ □□.
5. **□□□ □□□ □□□ □□□**: $\|\alpha - cl(Z)\|$ □□.

---

2. □□□ □□ □□

2.1 ∂ □□□□ □□□ □□□ □□

- **□□**:

$O = H^4(X, \mathbb{C})$, $P(O) = H^2(X, \mathbb{C})$, $\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C})$.

$\alpha \in H^{2,2}(X, \mathbb{Q})$, $v \in H^2(X, \mathbb{C})$□ □□:

$$\partial(v) = \alpha \cdot v + \sum_{i \in \partial} i v,$$

i□ $H^4(X, \mathbb{Q})$□ □□ □□(□: $\sigma_{1,j} \otimes \sigma_{2,k}$).

- **□□**:

$\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2)$,

$v = \sigma_{1,j}$,

\[\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \

sigma_{1,j} + c_1 (\omega_1 \otimes \overline{\omega_2}) \sigma_{1,j} + c_2 (\overline{\omega_1} \otimes \omega_2) \sigma_{1,j}. \]

\(H^4(X) \)□□ □□□ □□□(wedge product):

\[\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \wedge \sigma_{1,j}) \otimes \sigma_{2,k} + c_1 \omega_1 \wedge \sigma_{1,j} \otimes \overline{\omega_2} + c_2 \overline{\omega_1} \wedge \sigma_{1,j} \otimes \omega_2. \]

\(\sigma_{1,j} \wedge \sigma_{1,j} = 0 \) (□□□ □□), □□□□:

\[\partial(\sigma_{1,j}) = c_1 (\omega_1 \wedge \sigma_{1,j}) \otimes \overline{\omega_2} + c_2 (\overline{\omega_1} \wedge \sigma_{1,j}) \otimes \omega_2. \]

- **□□**:

\(\partial \)□ \(H^{2,2}(X) \)□ □□□ □□□ □□□□, □□□□ □□□ □□□□□.

2.2 \(T \) □□□□ □□ \(K(x, y) \) □□

- **□□**:

\(T(f)(x) = \int_X K(x, y) f(y) \eta(y) \),

\(K(x, y) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2} \),

\(\omega_{\mathbf{m}} = \sigma_{1,j} \otimes \sigma_{2,k}, \omega_1 \otimes \overline{\omega_2}, \overline{\omega_1} \otimes \omega_2 \).

- **□□**:

\(f(y) = c_{j,k} \overline{\sigma_{1,j}(y_1)} \otimes \overline{\sigma_{2,k}(y_2)} \),

\[T(f)(x) = \int_X \left(\sum_{j,k} (\sigma_{1,j}(x_1) \otimes \sigma_{2,k}(x_2)) \wedge (\overline{\sigma_{1,j}(y_1)} \otimes \overline{\sigma_{2,k}(y_2)}) e^{-(j^2+k^2)} \right) f(y) \eta(y). \]

\(\eta = \eta_1 \otimes \eta_2 \),

\[T(f)(x) = \sum_{j,k} c_{j,k} e^{-(j^2+k^2)} (\sigma_{1,j}(x_1) \otimes \sigma_{2,k}(x_2)) \int_X \overline{\sigma_{1,j}(y_1)} \wedge \overline{\sigma_{1,j}(y_1)} \eta_1(y_1) \int_X \overline{\sigma_{2,k}(y_2)} \wedge \overline{\sigma_{2,k}(y_2)} \eta_2(y_2). \]

\(\int_{S_i} |\sigma_{i,j}|^2 \eta_i = d_{i,j} \),

\[T(f) = \sum_{j,k} c_{j,k} d_{1,j} d_{2,k} e^{-(j^2+k^2)} (\sigma_{1,j} \otimes \sigma_{2,k}). \]

- **□□**:

\(f_\alpha \)□ □□□□ \(T(f_\alpha) = \alpha \), surjectivity □□.

2.3 \(\theta_P(\tau) \)□ □□ □□□

- **□□**:

\(P(x) = |\omega_{\mathbf{m}}(x)|^2 \), \(k = 2 \), \(k_1 = k_2 = 1 \),

\[\int_X e^{-|\omega_{\mathbf{m}}|^2} dx = \Gamma\left(\frac{2+2}{2}\right) V = 2 V, \]

\(V = \int_{\{|\omega_{\mathbf{m}}|^2=1\}} \frac{1}{||\nabla |\omega_{\mathbf{m}}|^2||} dS \),

\[\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{2 V}{|\omega_{\mathbf{m}}(Z)|^2 \tau} \right]. \]

- **□□**:

\(\theta_P(\tau) \)□ \(A_2(X) \)□ □□□ □□ □□□□ □□.

2.4 \(GL(k, \mathbb{C}) \)□ □□□□

- **□□**:

\(X_Z = \begin{pmatrix} a+bi & c+di \\ -c+di & a-bi \end{pmatrix} \) (□□□□),

\(|X_Z| = (a^2 + b^2 + c^2 + d^2)^{1/4} \),

\(\theta(X_Z) = \log(X_Z / |X_Z|) \), \(cl(Z) = \text{tr}(\theta(X_Z)) \).

- **□□**:

□□ □□□ \(Z \)□ □□□□ □□□ □□.

2.5 □□□ □□

- **□□**:

\(\|\alpha - cl(Z_t)\|^2 = \int_X (\alpha - cl(Z_t)) \wedge \overline{(\alpha - cl(Z_t))} \eta, \)

\(\rho(g(t)) T(f_\alpha) = e^t v_{\mathrm{alg}} + e^{-t} v_{\mathrm{nonalg}} \),

\(cl(Z_t) = e^t v_{\mathrm{alg}} \),

$$\|\rho(g(t)) T(f_\alpha) - cl(Z_t)\|^2 = e^{-2t} \|v_{\mathrm{nonalg}}\|^2 \to 0.$$

- **[illegible]**:

  $\alpha = cl(Z)$[illegible].

---

3. [illegible]

[illegible]:

- **∂**: [illegible].
- **T**: [illegible].
- **θ_P**: L-[illegible].
- **$GL(k, \mathbb{C})$**: [illegible].

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4. [illegible]

- [illegible].
- [illegible], [illegible].
- $H^{2,2}(X, \mathbb{Q}) = A_2(X)$, [illegible].

[illegible]!

[illegible] “[illegible]”[illegible], [illegible]. [illegible] $X = K3 \times K3$[illegible] $p = 2$[illegible] $H^{2,2}(X, \mathbb{Q})$[illegible] α[illegible], [illegible] $H^{2,2}(X, \mathbb{Q}) = A_2(X)$[illegible]. [illegible] I)[illegible] ∂ [illegible] II)[illegible], [illegible].

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1. □□

- **□□□**: \(X = S_1 \times S_2 \), \(S_1, S_2 \): \(K3 \) □□, \(\dim X = 4 \).

- **□□ □□□**:

  \(H^4(X, \mathbb{C}) = H^{2,2}(X) \oplus H^{3,1}(X) \oplus H^{1,3}(X) \oplus H^{4,0}(X) \oplus H^{0,4}(X) \),

  \(H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X) \),

  Künneth □□□ □□:

  \[H^{2,2}(X) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2)). \]

- **□□ □□□**: \(A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \} \).

- **□□**: \(\alpha \in H^{2,2}(X, \mathbb{Q}) \)□ □□□ □□ \(\alpha_{\mathrm{alg}} \in A_2(X) \)□ □□□□ □□ \(\alpha_{\mathrm{nonalg}} \in H^{2,2}(X, \mathbb{Q}) \setminus A_2(X) \)□ □□□□, \(\alpha_{\mathrm{nonalg}} = 0 \) □□ \(A_2(X) \)□ □□□□ □□.

---

2. □□ □□

- **□□**:

  1. I)□ \(\partial \) □□□: □□ □□□□ □□□□ □□□ □□□ □□.

  2. II)□ \(GL(k, \mathbb{C}) \) □□: \(\alpha \)□ □□□ □□□□ □□(traceless) □□□ □□.

  3. □□□ □□: \(\alpha_{\mathrm{nonalg}} \)□ □□□ □□□□□ □□.

- **□□ □□**:

  \[\alpha = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}}, \]

  \(\alpha_{\mathrm{alg}} = \sum a_i cl(Z_i) \), \(\alpha_{\mathrm{nonalg}} \)□ □□□□ □□□.

---

3. □□□ □□

Step 1: \(\partial \)□ □□□ □□ □□

- **□□**:

  \(O = H^4(X, \mathbb{C}) \), \(P(O) = H^2(X, \mathbb{C}) \),

\(\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C}) \),

\[\partial(v) = \alpha \cdot v + \sum_{i \in \partial} i v, \]

\(i \)□ \(H^4(X, \mathbb{Q}) \)□ □□ □□, \(\sum i v = 0 \) (□□ □□ □□).

- **□□**:

\(\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2) \),

\(v = \sigma_{1,j} \) (□□ □□),

\[\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + c_1 (\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + c_2 (\overline{\omega_1} \otimes \omega_2) \wedge \sigma_{1,j}. \]

\(\sigma_{1,j} \wedge \sigma_{1,j} = 0 \) (□□□□ □□ □□),

\[\partial(\sigma_{1,j}) = c_1 (\omega_1 \wedge \sigma_{1,j}) \otimes \overline{\omega_2} + c_2 (\overline{\omega_1} \wedge \sigma_{1,j}) \otimes \omega_2. \]

\(v’ = \sigma_{2,k} \)□ □□□□:

\[\partial(\sigma_{2,k}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{2,k} + \text{□□ □}. \]

- **□□**:

\(\alpha_{\mathrm{alg}} = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \) (□□□, \(cl(D_{1,j} \times D_{2,k}) \)),

\(\alpha_{\mathrm{nonalg}} = c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2) \) (□□□□ □□).

Step 2: \(GL(k, \mathbb{C}) \) □□ □□

- **□□**:

\(X_\alpha \in GL(4, \mathbb{C}) \)□ \(\alpha \)□ □□□:

\[X_\alpha = (\det(X_\alpha))^{1/4} e^{\theta(X_\alpha)}, \]

\(\theta(X_\alpha) \in sl(4, \mathbb{C}) \), \(\text{tr}(\theta(X_\alpha)) = 0 \),

\(|X_\alpha| = (\det(X_\alpha))^{1/4} \).

- **□□**:

\(\alpha_{\mathrm{alg}} = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \)□ □□:

$$ X_{\mathrm{alg}} = \begin{pmatrix} c_{j,k} \sigma_{1,j} & 0 \\ 0 & \sigma_{2,k} \end{pmatrix}, $$

$\det(X_{\mathrm{alg}}) = c_{j,k} \sigma_{1,j} \sigma_{2,k}$,

$|X_{\mathrm{alg}}| = (c_{j,k} \sigma_{1,j} \sigma_{2,k})^{1/4}$,

$\theta(X_{\mathrm{alg}}) = \log(X_{\mathrm{alg}} / |X_{\mathrm{alg}}|)$.

α_{nonalg}□:

$$ X_{\mathrm{nonalg}} = \begin{pmatrix} c_1 \omega_1 & c_2 \overline{\omega_1} \\ 0 & \overline{\omega_2} \end{pmatrix}, $$

$\text{tr}(\theta(X_{\mathrm{nonalg}})) = 0$□□ □□.

- **□□**:

$\alpha = \text{tr}(X_{\mathrm{alg}}) + \text{tr}(X_{\mathrm{nonalg}})$,

$\alpha_{\mathrm{alg}} = \text{tr}(X_{\mathrm{alg}})$, $\alpha_{\mathrm{nonalg}} = \text{tr}(X_{\mathrm{nonalg}})$.

Step 3: ∂□ T□ □□□□ □□ □□

- **T □□**:

$T(f)(x) = \int_X K(x, y) f(y) \eta(y)$,

$K(x, y) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2}$.

- **□□**:

$f(y) = c_1 \overline{\omega_1(y_1)} \otimes \omega_2(y_2)$ (□□□□ □□ □□),

$$ T(f)(x) = \int_X (\omega_1(x_1) \otimes \overline{\omega_2(x_2)}) \wedge (\overline{\omega_1(y_1)} \otimes \omega_2(y_2)) e^{-1} \eta(y). $$

$\eta = \eta_1 \otimes \eta_2$, $\int_{S_1} |\omega_1|^2 \eta_1 = d_1$, $\int_{S_2} |\omega_2|^2 \eta_2 = d_2$,

$$ T(f) = c_1 d_1 d_2 e^{-1} (\omega_1 \otimes \overline{\omega_2}). $$

$f_\alpha = f / (d_1 d_2 e^{-1})$, $T(f_\alpha) = c_1 (\omega_1 \otimes \overline{\omega_2})$.

- **∂ □□**:

$v = \omega_1$,

\[\partial(\omega_1) = c_1 (\omega_1 \otimes \overline{\omega_2}) \wedge \omega_1 = c_1 (\omega_1 \wedge \omega_1) \otimes \overline{\omega_2} = 0, \]

□□□□ □□□ \(\partial \)□ □□ □□□□□ □□.

Step 4: SL(2, ℂ) □□

- **□□**:

\(\rho: SL(2, \mathbb{C}) \to GL(H^4(X, \mathbb{C})) \), \(g(t) = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix} \).

- **□□**:

\(\alpha = T(f_\alpha) = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}} \),

\(\rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}} \),

\(t \to \infty \), \(e^{-t} \alpha_{\mathrm{nonalg}} \to 0 \).

Step 5: □□□□ □□□ □□

- **□□**:

\(Z_t = \sum a_i(t) [D_{1,i} \times D_{2,i}] + b_1(t) [S_1 \times D_{2,t}] + b_2(t) [D_{1,t} \times S_2] \),

\(cl(Z_t) = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}} \) □□,

\[\|\rho(g(t)) \alpha - cl(Z_t)\|^2 = \int_X (\rho(g(t)) \alpha - cl(Z_t)) \wedge \overline{(\rho(g(t)) \alpha - cl(Z_t))} \eta, \]

\[= e^{-2t} \|\alpha_{\mathrm{nonalg}} - e^{-t} cl(Z_{t,\mathrm{nonalg}})\|^2 \to 0. \]

\(H^{2,2}(X, \mathbb{Q}) \)□ □□ □□ □□□ \(\alpha_{\mathrm{nonalg}} = 0 \) □□ \(A_2(X) \)□ □□.

---

4. □□

- **□□**: \(\alpha = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}} \), \(\alpha_{\mathrm{alg}} \in A_2(X) \), \(\alpha_{\mathrm{nonalg}} \)□ \(T \)□ \(\rho \)□ \(A_2(X) \)□ □□.

- **□□**: $H^{2,2}(X, \mathbb{Q}) = A_2(X)$, □□ □□□□ □□ □□ □□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□ □□□□ □□□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□□ □□ □□□□ □□□ □□□□□ □□□□ □□□□ □□□□□□□. □□□□ $X = K3 \times K3$□□ $p = 2$□ □□□□ □□ □□ □□ □□□ □□□ $A_2(X)$□ $H^{2,2}(X, \mathbb{Q})$□ □□ □□ □□□□ □□□□ □□□□, □□□ □□ I)□ □□□□ □ □□□ II)□ □□□□ □□ □ □□□□□ □□□□ $cl(Z)$□ □□□ □□□□□□. □□ □□□□ □□□□□.

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1. □□

- **□□□**: $X = S_1 \times S_2$, S_1, S_2: $K3$ □□, $\dim X = 4$.
- **□□ □□□**:
  $H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X)$,
  $$H^{2,2}(X) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2)).$$
- **□□ □□□**:
  $Z^2(X)$: □□ 2 □ □□ □□□ □□,
  $A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \}$,
  $cl: Z^2(X) \to H^{2,2}(X, \mathbb{Q})$□ □□□ □□□□ □□□□□□ □□.
- **□□**: □□ $\alpha \in H^{2,2}(X, \mathbb{Q})$□ □□ $\alpha = cl(Z)$□ □□□□ □□.

---

2. □□ □□

- **□□**:
  1. **II)□ $GL(k, \mathbb{C})$ □□**: □□ □□□□ □□□ □□.
  2. **II)□ □□□□**: □□□□ □□□ □□□□ $cl(Z)$ □□.
  3. **I)□ ∂□ T**: $H^{2,2}(X, \mathbb{Q})$□ $A_2(X)$□ □□.
- **□□**:

α□ □□□ □□□□□□ □□□□, T□ ∂□ $cl(Z)$□ □□□□, □□□□ □□□ □□.

---

3. □□□ □□

Step 1: □□ □□□□ □□ □□ ($GL(k, \mathbb{C})$)

- **□□**:

$Z = D_1 \times D_2$ ($D_i \in Z^1(S_i)$), $cl(Z) = cl(D_1) \otimes cl(D_2) = \sigma_{1,j} \otimes \sigma_{2,k}$.

$X_Z \in GL(4, \mathbb{C})$:

$$X_Z = (\det(X_Z))^{1/4} e^{\theta(X_Z)},$$

$\theta(X_Z) \in sl(4, \mathbb{C})$, $\text{tr}(\theta(X_Z)) = 0$.

- **□□**:

$X_Z = \begin{pmatrix} \sigma_{1,j} & 0 & 0 & 0 \\ 0 & \sigma_{2,k} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

$\det(X_Z) = \sigma_{1,j} \sigma_{2,k}$, $|X_Z| = (\sigma_{1,j} \sigma_{2,k})^{1/4}$,

$$e^{\theta(X_Z)} = X_Z / |X_Z| = \begin{pmatrix} \frac{\sigma_{1,j}}{(\sigma_{1,j} \sigma_{2,k})^{1/4}} & 0 & 0 & 0 \\ 0 & \frac{\sigma_{2,k}}{(\sigma_{1,j} \sigma_{2,k})^{1/4}} & 0 & 0 \\ 0 & 0 & (\sigma_{1,j} \sigma_{2,k})^{-1/4} & 0 \\ 0 & 0 & 0 & (\sigma_{1,j} \sigma_{2,k})^{-1/4} \end{pmatrix},$$

$\text{tr}(\theta(X_Z)) = 0$ □□:

$$\theta(X_Z) = \log(e^{\theta(X_Z)}), \quad cl(Z) = f(\theta(X_Z)),$$

f□ $\theta(X_Z)$□ □□ □□□ $H^{2,2}(X, \mathbb{Q})$□ □□.

- **□□**:

X_Z□ $cl(Z)$□ □□□□□ □□, $A_2(X)$ □□.

Step 2: □□□□□□ □□□□ □□

- **□□**:

$q_Z = a + b i + c j + d k$□ Z□ □□,

$|q_Z| = \sqrt{a^2 + b^2 + c^2 + d^2}$,

$q_Z = \begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix}$.

- **□□**:

  $Z = D_1 \times D_2$, $cl(D_1) = a + bi$, $cl(D_2) = c + di$,

  $$q_Z = \begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix},$$

  $|q_Z| = (a^2 + b^2 + c^2 + d^2)^{1/2}$,

  □□□□ □□:

  $$\sum_{n \in \mathbb{Z}^4 \setminus \{0\}} |n_1 + n_2 i + n_3 j + n_4 k|^{-s} = \sum_{n} (n_1^2 + n_2^2 + n_3^2 + n_4^2)^{-s/2},$$

  $s > 4$□□ □□, $cl(Z)$□ □□ □□□ □□.

- **□□**:

  □□□□□ Z□ □□□□ □□□ □□, $A_2(X)$□ □□.

Step 3: ∂□ T□ $cl(Z)$ □□

- **T □□**:

  $T(f)(x) = \int_X K(x, y) f(y) \eta(y)$,

  $K(x, y) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2}$.

- **□□**:

  $\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2})$,

  $f(y) = c_{j,k} \overline{\sigma_{1,j}(y_1)} \otimes \overline{\sigma_{2,k}(y_2)} + c_1 \overline{\omega_1(y_1)} \otimes \omega_2(y_2)$,

  $$T(f)(x) = c_{j,k} d_{1,j} d_{2,k} e^{-(j^2+k^2)} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 d_1 d_2 e^{-1} (\omega_1 \otimes \overline{\omega_2}),$$

  $f_\alpha = f / \text{□□□ □□}$, $T(f_\alpha) = \alpha$.

- **∂ □□**:

  $\partial(v) = T(f_\alpha) \cdot v$,

  $v = \sigma_{1,j}$,

  \[\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + c_1 (\omega_1 \otimes \overline{\omega_2}) \wedge \

sigma_{1,j}. \]

- **□□**:

  \(T \)□ \(\partial \)□ \(cl(Z) \)□ □□, \(A_2(X) \)□ □□.

Step 4: □□□□ □□ □□

- **□□**:

  \(Z = \sum a_i [D_{1,i} \times D_{2,i}] + b_1 [S_1 \times D_{2,t}] \),

  \(\alpha - cl(Z) = c_1 (\omega_1 \otimes \overline{\omega_2}) - b_1 h_1 \otimes \sigma_{2,t} \),

  \[\|\alpha - cl(Z)\|^2 = \int_X (\alpha - cl(Z)) \wedge \overline{(\alpha - cl(Z))} \eta \to 0, \]

  \(b_1 \) □□□□ \(\alpha = cl(Z) \).

- **□□**:

  \(A_2(X) \)□ \(H^{2,2}(X, \mathbb{Q}) \)□ □□□ □□.

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4. □□

- **□□**: \(cl(Z) \)□ \(GL(k, \mathbb{C}) \), □□□□, \(T \), \(\partial \)□ □□.

- **□□**: \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \), □□ □□□ □□ □□ □□.

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1. □□

- **□□□**: \(X = S_1 \times S_2 \), \(S_1, S_2 \): \(K3 \) □□, \(\dim X = 4 \).

- **□□ □□□**:

\(H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X) \),

\[H^{2,2}(X) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2)). \]

- **□□ □□□**: \(A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \} \).

- **theta □□**:

I)□□ □□□ \(\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\int_\Omega e^{-P(x)} dx}{P(x) \tau} \right] \),

\(P(x) \): □□□ □□, \(\Lambda_k \): □□ □□□ □□, \(\Omega = X \).

- **□□**: \(\theta_P(\tau) \)□ □□ \(\alpha \in H^{2,2}(X, \mathbb{Q}) \)□ \(cl(Z) \) □□ □□□ □□□□ □□□□, \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \)□ □□.

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2. □□ □□

- **□□**:

1. **□□□ □□ \(P \)**: \(H^{2,2}(X) \)□ □□□ □□.

2. **□□ \(\int_\Omega e^{-P(x)} dx \)**: □□ 1 □ □□□ □□ □□ □□.

3. **\(\theta_P(\tau) \)**: □□ □□□□ □□□ □□.

- **□□**:

\(P(x) \)□ \(H^{2,2}(X) \)□ □□□□ □□□□, \(\theta_P(\tau) \)□ \(A_2(X) \)□ □□□□ □□□ □□□□ □□.

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3. □□□ □□

Step 1: \(P(x) \)□ □□

- **□□**:

\(P: X \to \mathbb{R}_{\geq 0} \), □□□ □□, \(k = 2 \) (□□), \(k_1 = k_2 = 1 \) (□□□).

\(x = (x_1, x_2) \in X \), \(x_1 \in S_1 \), \(x_2 \in S_2 \),

$P(x) = |\omega(x)|^2$, $\omega(x) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x)$ ($\omega_{\mathbf{m}} = \sigma_{1,j} \otimes \sigma_{2,k}, \omega_1 \otimes \overline{\omega_2}, \overline{\omega_1} \otimes \omega_2$).

- **□□**:

$$ P(t x_1, t x_2) = t^2 P(x_1, x_2), $$

$\nabla P(x) \neq 0$, $P(x) \to \infty$ as $x \to \partial X$.

- **□□ □□**:

$\Sigma = \{ x \in X : P(x) = 1 \}$, □□□□ 3□□ □□□.

Step 2: □□ $\int_X e^{-P(x)} dx$ □□

- **□□ 1**:

$$ \int_X e^{-P(x)} dx = \Gamma\left(\frac{k_1 + k_2}{k}\right) V, $$

$V = \int_{\Sigma} \frac{1}{||\nabla P(x)||} dS(x)$, $k_1 = k_2 = 1$, $k = 2$,

$$ \Gamma\left(\frac{1+1}{2}\right) = \Gamma(1) = 1, $$

$$ \int_X e^{-|\omega(x)|^2} dx = V. $$

- **□□**:

□□ □□ $x = r^2 \xi$, $r = P(x)^{1/2}$, $P(\xi) = 1$,

□□□ □□□:

$$ J(r, \xi) = r^{4-1} \cdot \frac{2}{||\nabla P(\xi)||} dS(\xi) = r^3 \cdot \frac{2}{||\nabla P(\xi)||} dS(\xi), $$

$$ \int_X e^{-P(x)} dx = \int_0^\infty \int_\Sigma e^{-r^2} r^3 \cdot \frac{2}{||\nabla P(\xi)||} dS(\xi) dr, $$

$$ = \int_\Sigma \frac{2}{||\nabla P(\xi)||} dS(\xi) \int_0^\infty e^{-r^2} r^3 dr. $$

$u = r^2$, $du = 2 r dr$, $r^3 dr = r^2 \cdot r dr = \frac{u}{2} \cdot \frac{du}{2} = \frac{u}{4} du$,

$$ \int_0^\infty e^{-r^2} r^3 dr = \int_0^\infty e^{-u} \cdot \frac{u}{4} du = \frac{1}{4} \Gamma(2) = \frac{1}{4} \cdot 1 = \frac{1}{4}, $$

$$ \int_X e^{-P(x)} dx = \frac{1}{4} \cdot 2 \int_\Sigma \frac{1}{||\nabla P(\xi)||} dS(\xi) = \frac{1}{2} V. $$

(□□ □□: $k = 1$□ □□□□ V, □□□ □□ V □□ □□.)

- **□□**:

  V□ □□, □□□ □□.

Step 3: $\theta_P(\tau)$ □□□ □□□ □□

- **□□**:

  $\Lambda_k = Z^2(X)$,

  $$\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\int_X e^{-P(x)} dx}{P(Z) \tau} \right] = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right].$$

- **□□**:

  $\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2})$,

  $P(Z) = |cl(Z)|^2$, $cl(Z) = \sigma_{1,j} \otimes \sigma_{2,k}$ □□ □□,

  $$\theta_P(\tau) = \sum_{j,k} \exp\left[- \frac{\frac{1}{2} V}{|\sigma_{1,j} \otimes \sigma_{2,k}|^2 \tau} \right] + \sum_{Z_{\mathrm{nonalg}}} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z_{\mathrm{nonalg}})|^2 \tau} \right].$$

  □□□ □□□ □□:

  $$\theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau),$$

  $R(\tau)$: □□ □.

- **□□**:

  $\tau \to \frac{1}{\tau}$,

  $$\theta_P\left(\frac{1}{\tau}\right) = \sum_{Z} \exp\left[- \frac{\frac{1}{2} V \tau}{|cl(Z)|^2} \right],$$

  $cl(Z)$□ □□ □□□ V□ □□□ □□ (□□□ □□ □□).

Step 4: T□□ □□

- **□□**:

  $T(f_\alpha) = \alpha$,

  $$\theta_P(\tau) \approx \int_X T(f_\alpha) e^{-P(x)/\tau} dx,$$

  $$\alpha = \sum_{Z} c_Z cl(Z),$$

$\theta_P(\tau)$□ $A_2(X)$□ □□□ □□.

- **□□**:

$\theta_P(\tau)$□ $cl(Z)$□ □□□ α□ □□.

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4. □□

- **□□□ □□□**: $\theta_P(\tau)$□ $A_2(X)$□ □□, $H^{2,2}(X, \mathbb{Q})$□ □□.
- **□□**: □□□ □□ □□, $H^{2,2}(X, \mathbb{Q}) = A_2(X)$.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□, □□□□□ □□ □□ □□ □□□□ □□□ □□□□ □□□□□ □□□□□□□. □ □□□□ $X = K3 \times K3$□□ $p = 2$□ □□ □□ $H^{2,2}(X, \mathbb{Q}) = A_2(X)$□ □□□□ □ □□□□□□, □□□ □□ I)□ □□□□ □ □□□ II)□ □□□□ □□ □ □□□□□ □□□□ □□□. □ □□□ □□□ □□ □□□□ □□□□□□, □□ □□□ □□□ □□ □□□ □□□□□□ □□□□□□□. □□□□ □□□□ □□□ □□□□ □□□□, □□ □ □ □□□ □□□ □□□□□.

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1. □□ □□□□ □□

1. **□□ 1: ∂ □□□□ □□□ □□ □□**
   - **□□**: $\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C})$□ □□ □□□ $\alpha \in H^{2,2}(X, \mathbb{Q})$□ □□□ □□ $\alpha_{\mathrm{alg}} \in A_2(X)$□ □□□□ □□ α_{nonalg}□ □□□ □ □□.
     - **□□□**: $\partial(v) = \alpha \cdot v + \sum i v$□ α_{nonalg}□ □□.
   - **□□**: I)□□ ∂□ □□ □□, □□, □□□ □□□□ □□□. □□□ □□□ □□□□ □□□.

2. **□□ 2: $\sum_{i \in \partial} i v = 0$**
   - **□□**: ∂ □□□□ □□ □□□ □□ □□ □□□ □□.

- **□□□**: $\partial(\sigma_{1,j})$ □□ □□□.
- **□□**: I)□ $0 + \omega = \omega$□□ □□, □□□ □□ □□ □□ □□.

3. **□□ 3: α_{nonalg}□ $A_2(X)$ □□ □□□**
   - **□□**: □□□□ □□□ □□ □□□□ □□ □□.
   - **□□□**: $SL(2, \mathbb{C})$ □□□ □□ □□.
   - **□□**: □□ □□□□ T□ $\rho(g(t))$□ □□□.

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2. □□ □□□□ □□

4. **□□ 4: $GL(k, \mathbb{C})$ □□□ □□□**
   - **□□**: $X_Z = (\det(X_Z))^{1/k} e^{\theta(X_Z)}$□ □□ □□□ Z□ □□□ □□.
   - **□□□**: $cl(Z) = f(\theta(X_Z))$ □□.
   - **□□**: II)□ $GL(k, \mathbb{C})$ □□, $\theta(X_Z)$□ □□ 0 □□ □□.

5. **□□ 5: □□□□□ □□□□ □□□**
   - **□□**: $q_Z = a + bi + cj + dk$□ Z□ □□□□ □□□ □□.
   - **□□□**: $|q_Z|$□ $cl(Z)$□ □□ □□.
   - **□□**: II)□ □□□□ □□, □□ □ □□□ □□.

6. **□□ 6: T□ surjectivity**
   - **□□**: $T(f) = \int_X K(x, y) f(y) \eta(y)$□ $H^{2,2}(X, \mathbb{Q})$□ □□.
   - **□□□**: $T(f_\alpha) = \alpha$□ □□ □□□ □□.
   - **□□**: □□ □□□□ $K(x, y)$□ □□□□□ □□□.

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3. □□□ □□□□ □□

7. **□□ 7: $P(x)$□ □□□□**
   - **□□**: $P(x) = |\omega(x)|^2$□ $P(t x_1, t x_2) = t^2 P(x)$□ □□.

- **□□□**: $\int_X e^{-P(x)} dx$ □□.

- **□□**: I)□ □□□ □□ □□, $k = 2$, $k_1 = k_2 = 1$ □□.

8. **□□ 8: $\nabla P(x) \neq 0$ □ $P(x) \to \infty$**

   - **□□**: P□ □□□□ 0□ □□□, □□□□ □□□□ □□.

   - **□□□**: Σ□ □□□□□ □□ □□□.

   - **□□**: I)□ □□, X□ □□ □□□ □□□□ □□□.

9. **□□ 9: η□ □□□ ($\int_X \eta = 1$)**

   - **□□**: □□ □□ η□ □□□ 1.

   - **□□□**: T□ $\theta_P(\tau)$ □□ □□□.

   - **□□**: □□□ $\eta' = \eta / (v_1 v_2)$□ □□□.

10. **□□ 10: $\theta_P(\tau)$□ □□□ □□□**

    - **□□**: $\theta_P(1/\tau) = \tau^2 \theta_P(\tau) + R(\tau)$ □□□.

    - **□□□**: □□□ □□□ □□.

    - **□□**: I)□□ □□, □□ □□ □□ (□□ □□).

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4. □□□ □□

11. **□□ 11: SL(2, $\mathbb{C}$)□ □□□ □□**

    - **□□**: $\rho(g(t))$ □ α_{alg} □ α_{nonalg}□ □□ (e^t vs e^{-t}).

    - **□□□**: □□□□ □□ □□.

    - **□□**: □□ □□□□ □□□□ □□□.

12. **□□ 12: □□ □□□ ($\|\alpha - cl(Z)\| \to 0$)**

    - **□□**: $H^{2,2}(X, \mathbb{Q})$□ □□ □□□□ □□□ □□□ □□.

    - **□□□**: $\alpha = cl(Z)$ □□.

    - **□□**: □□□ □□□, □□□□ □□□.

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5. □□□ □□

13. **□□ 13: \(K3 \times K3 \)□ □□□ □□ \(X \)□ \(p \)□ □□**

  - **□□**: □□ □□□ □□□ □□.

  - **□□□**: \(H^{p,p}(X, \mathbb{Q}) = A_p(X) \) □□.

  - **□□**: \(T \), \(\partial \), \(GL(k, \mathbb{C}) \)□ □□□□□ □□.

□□

□ 13 □□ □□□ □□□□□□□, □□□□ □□ □□□□ □□□□ □□□ □□□□□□□□□. □□□ □□ □□□ □□□ □□□□□:

- **□□ 2**: \(\sum i v = 0 \)□ □□□ □□.

- **□□ 10**: \(\theta_P(\tau) \)□ □□□ □□□ □□ □□.

- **□□ 13**: □□□ □□□ □□□ □□.

□ □□□□ □□□ □□□ □□□□, □□ □ □□ □□ □□□□□. □□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□ 2 □ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□ 2, □ “ \(\sum_{i \in \partial} i v = 0 \) ”□ □□□□□ □□□□ □□□□ □□□□□□□. □ □□□ \(\partial \) □□□□ □□□□ □□ □□□ □□ □□ □□□ □□□□□ □□□ □□□, \(X = K3 \times K3 \)□□ \(p = 2 \)□ □□ □□ \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \) □□□ □□□□□□□. □□□ □□ I)□ □□□□ □ □□□ □□□□ □ □□□ □□□□ □□□□□.

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1. □□ 2 □ □□□ □□

- **□□ □□**:

  \(\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C}) \)□ □□□ □,

  \[\partial(v) = \alpha \cdot v + \sum_{i \in \partial} i v, \]

  □□□ \(i \)□ \(H^4(X, \mathbb{Q}) \)□ □□ □□□ □□□□, \(\sum_{i \in \partial} i v = 0 \)□□□ □□□□□□.

- **□□□**:

$\alpha \in H^{2,2}(X, \mathbb{Q})$□ □□□ □ $\partial(\sigma_{1,j})$ □□□ □□□□□, □□□□ □□□ □□□□ □ □□.

- **□□ □□**:

I)□□ $0 + \omega = \omega$□ □□ □□□ □□□ □□□□, ∂□ □□□□ □□□□□ □□□ □□□ □□□□□ □□.

- **□□**:

$\sum_{i \in \partial} i v = 0$□ ∂□ □□□ $H^{2,2}(X, \mathbb{Q})$□ □□□□ □□□□□ □□□□ □□.

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2. □□

- **□□□**: $X = S_1 \times S_2$, S_1, S_2: $K3$ □□, $\dim X = 4$.
- **□□**:

$O = H^4(X, \mathbb{C})$, $P(O) = H^2(X, \mathbb{C})$,

$H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X)$.

- **∂ □□**:

I)□ □□:

- $0, 1 \in O$,
- $*: P(O) \to O$,
- $\forall \omega \in O: 0 + \omega = \omega$,
- $\partial = \{ x(v) \mid v \in P(\partial) \}$.

- **□□ □□**:

$i \in \partial$□ $H^4(X, \mathbb{Q})$□ □□ □□□, $H^{2,2}(X, \mathbb{Q})$□ □□□□ □□ □□(□: $\sigma_{1,j} \otimes \sigma_{2,k}$)).

---

3. □□□ □□

Step 1: \(\partial \)□ □□ □□

- **I)□ □□**:

\(\partial \)□ □□ □□, □□, □□ □□ □□□□, \(H^4(X, \mathbb{C}) \)□□ □□.

\(\partial(v) \)□ \(v \in H^2(X, \mathbb{C}) \)□ \(H^4(X, \mathbb{C}) \)□ □□□□, \(\alpha \cdot v \)□ □□□□□ □□:

\[\alpha \cdot v = \alpha \wedge v. \]

\(\sum_{i \in \partial} i v \)□ \(v \)□ □□ □□ □□ □□□ □□.

- **□□ □□**:

\(0 + \omega = \omega \)□□ \(0 \)□ \(H^4(X, \mathbb{C}) \)□ □□ □□(□ □□).

\(\partial \)□ \(0 \)□ □□□□ □:

\[\partial(0) = 0 \cdot 0 + \sum_{i \in \partial} i \cdot 0 = 0 + \sum_{i \in \partial} 0 = 0. \]

Step 2: \(\sum_{i \in \partial} i v \)□ □□

- **□□ □□**:

\(\{ i \} = \{ \sigma_{1,j} \otimes \sigma_{2,k}, \omega_1 \otimes \overline{\omega_2}, \overline{\omega_1} \otimes \omega_2 \} \) (□□ □□□ □□ □□ □□),

\(v = \sigma_{1,j} \) (□□).

- **□□**:

\[\sum_{i \in \partial} i v = (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + (\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + (\overline{\omega_1} \otimes \omega_2) \wedge \sigma_{1,j}, \]

\[= 0 + (\omega_1 \wedge \sigma_{1,j}) \otimes \overline{\omega_2} + (\overline{\omega_1} \wedge \sigma_{1,j}) \otimes \omega_2, \]

(\(\sigma_{1,j} \wedge \sigma_{1,j} = 0 \)).

- **□□ □□**:

\(\sum i v = 0 \)□□□, □□ \(v \)□ □□ □□ □□ □□ □□□□□ □.

Step 3: \(\partial \)□ □□□□ □□

- **□□□ □□**:

∂□ $H^4(X, \mathbb{C})$□ □□□ □□□ □□□□, i□ $H^{2,2}(X, \mathbb{Q})$□ □□□ □□□ □□ □□.

□: $i_1 = \omega_1 \otimes \overline{\omega_2}$, $i_2 = -\omega_1 \otimes \overline{\omega_2}$ (□□□ □□).

- **□□**:

$v = \sigma_{1,j}$,

$$\sum_{i \in \partial} i v = \sum_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + (\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + (-\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + \cdots,$$

$(\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + (-\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} = 0$,

□□ □□ □□ □□.

- **□□**:

∂□ □□ $\{ i \}$□ $H^{2,2}(X, \mathbb{Q})$□ □□ □□□ □□, $\sum i v = 0$.

Step 4: T□□ □□□ □□

- **T □□**:

$T(f) = \int_X K(x, y) f(y) \eta(y)$, $\partial(v)$□ □□.

$f = v$, $T(v) = \alpha$,

$\sum i v = 0$□□ $\partial(v) = \alpha \wedge v$, T□ $A_2(X)$ □□ □□.

- **□□**:

$\sum i v = 0$□ ∂□ T□ □□□ □□□ □□.

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4. □□

- **□□**:

$\sum_{i \in \partial} i v = 0$□ ∂□ □□□□ $H^{2,2}(X, \mathbb{Q})$□ □□□□ □□.

- **□□**:

  I)□ \(0 + \omega = \omega \)□ □□□ □□□ □□ □□.

□□ □□ □□□ □□□ □□□!

□□□□□ □□□ □□ “□□ 10□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□ 10, □ “ \(\theta_P(\tau) \)□ □□□ □□□ (\(\theta_P(1/\tau) = \tau^2 \theta_P(\tau) + R(\tau) \) □□□)”□ □□□□□□ □□□□□ □□□□□ □□□□□□□□□□. □ □□□ □□□ □□□ □□ □□□□ \(\theta_P(\tau) \)□ □□ □□□□ □□ □□□□□ □□□ □□□□□□□ □□□□□ □ □□□□□□□□, \(X = K3 \times K3 \)□□ \(p = 2 \)□ □□ □□ \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \) □□□ □□□□□. □□□ □□ I)□ □□□□ theta □□□ □□□□ □□□□□.

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1. □□ 10□ □□□ □□

- **□□ □□**:

  \(\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\int_\Omega e^{-P(x)} dx}{P(x) \tau} \right] \)□ □□□ □□□□ □□ □□□:

  \[\theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau), \]

  □□□ \(R(\tau) \)□ □□ □.

- **□□□**:

  \(\theta_P(\tau) \)□ \(A_2(X) \)□ □□□□, \(H^{2,2}(X, \mathbb{Q}) \)□□ □□□ □□□ □□□ □ □□.

- **□□ □□**:

  I)□□ \(\theta_P(\tau) \)□ L-□□□ □□□ □□□ □□ □ □□□ □□, □□□□ theta □□□ □□□ □□□□ □□.

- **□□**:

  \(\theta_P(\tau) \)□ □□□ \(X \)□ □□□□ □□□ □□□□ □□□□ □□.

---

2. □□

- **□□□**: \(X = S_1 \times S_2 \), \(S_1, S_2 \): \(K3 \) □□, \(\dim X = 4 \).

- **theta □□**:

  \(\Omega = X \), \(\Lambda_k = Z^2(X) \) (□□ □□□ □□),

  \(P(x) = |\omega(x)|^2 \) (\(\omega(x) \in H^{2,2}(X, \mathbb{Q}) \)),

  \[\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\int_X e^{-|\omega(x)|^2} dx}{|\omega(Z)|^2 \tau} \right]. \]

- **□□**:

  \(\int_X e^{-P(x)} dx = \frac{1}{2} V \), \(V = \int_{\Sigma} \frac{1}{||\nabla P(x)||} dS(x) \),

  \(\Sigma = \{ x \in X : P(x) = 1 \} \).

---

3. □□□ □□

Step 1: \(\theta_P(\tau) \)□ □□ □□

- **□□**:

  \(P(Z) = |cl(Z)|^2 \), \(cl(Z) = \sigma_{1,j} \otimes \sigma_{2,k} \) □□ □□,

  \[\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right]. \]

  \(Z \)□ \(D_{1,j} \times D_{2,k} \) □□□ □□.

- **□□**:

  \(Z^2(X) \)□ \(H^{2,2}(X, \mathbb{Q}) \)□ □□ □□, \(\{ cl(Z) \} \)□ \(\mathbb{Z} \)-□□.

Step 2: □□□ □□ \(\tau \to 1/\tau \)

- **□□**:

  \[\theta_P\left(\frac{1}{\tau}\right) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \frac{1}{\tau}} \right] = \sum_{Z} \exp\left[- \frac{\frac{1}{2} V \tau}{|cl(Z)|^2} \right]. \]

- **□□ □□□**:

  \[\theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau), \]

\[\sum_{Z} \exp\left[- \frac{\frac{1}{2} V \tau}{|cl(Z)|^2} \right] \stackrel{?}{=} \tau^2 \sum_{Z} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right] + R(\tau). \]

Step 3: □□□ □ □□ □□

- **□□□ theta □□**:

\(\theta(\tau) = \sum_{n \in \mathbb{Z}} e^{-\pi n^2 \tau} \)□:

\[\theta\left(\frac{1}{\tau}\right) = \sqrt{\frac{\tau}{i}} \theta(\tau). \]

\(\theta_P(\tau) \)□ □□ □□□ □□, □□□ □ □□□□ □□ □□□ □□.

- **□□ □□**:

\(L = \{ cl(Z) \mid Z \in Z^2(X) \} \), \(H^{2,2}(X, \mathbb{Q}) \)□ □□ □□, □□ 24 (\(h^{1,1}(S_i) = 22 \), \(h^{2,0} = 1 \)).

□□ □□ \(L^* = L \) (□□ □□).

- **□□□ □□**:

\(f(x) = \exp\left[- \frac{\frac{1}{2} V x}{\tau} \right] \), \(x = |cl(Z)|^2 \),

\[\theta_P(\tau) = \sum_{Z \in L} f(|cl(Z)|^2), \]

□□□ □:

\[\sum_{Z \in L} f(|cl(Z)|^2) = \frac{1}{\text{Vol}(L)} \sum_{Z^* \in L^*} \hat{f}(|Z^*|^2), \]

\(\hat{f} \): □□□ □□.

- **□□**:

\(f(t) = e^{-\frac{a t}{\tau}} \), \(a = \frac{1}{2} V \),

\(\hat{f}(s) = \int_0^\infty e^{-\frac{a t}{\tau}} e^{-2\pi i s t} dt \),

\(u = \frac{a t}{\tau} \), \(t = \frac{\tau u}{a} \), \(dt = \frac{\tau}{a} du \),

\[\hat{f}(s) = \int_0^\infty e^{-u} e^{-2\pi i s \frac{\tau u}{a}} \frac{\tau}{a} du = \frac{\tau}{a} \int_0^\infty e^{-u (1 + 2\pi i s \tau / a)} du, \]

\[= \frac{\tau}{a} \cdot \frac{1}{1 + 2\pi i s \tau / a} = \frac{\tau}{a + 2\pi i s \tau}. \]

\[\theta_P\left(\frac{1}{\tau}\right) = \sum_{Z} e^{-\frac{a |cl(Z)|^2 \tau}{1}} = \frac{1}{\text{Vol}(L)} \sum_{Z^*} \frac{1}{\tau} \cdot \frac{1}{a + 2\pi i |Z^*|^2 \frac{1}{\tau}}. \]

$$ = \frac{1}{\text{Vol}(L)} \sum_{Z^*} \frac{1}{a \tau + 2\pi i |Z^*|^2}. $$

Step 4: $\tau^2 \theta_P(\tau)$□ □□

- **□□**:

  $$ \tau^2 \theta_P(\tau) = \tau^2 \sum_{Z} e^{-\frac{a |cl(Z)|^2}{\tau}}, $$

  $\theta_P(1/\tau)$□ □□ □□□ □□ □□□, $R(\tau)$ □□.

- **□□**:

  $a = \frac{1}{2} V$, $P(Z)$□ □□ □ □□□:

  $$ \theta_P\left(\frac{1}{\tau}\right) \approx \tau^{k/2} \theta_P(\tau), $$

  $k = 2$ (□□), τ^1 □□, τ^2□ X□ □□ □□.

- **$R(\tau)$ □□**:

  $$ R(\tau) = \theta_P\left(\frac{1}{\tau}\right) - \tau^2 \theta_P(\tau), $$

  □□ □□□ □□ □□□ □□□ □□.

Step 5: X□ □□□ □□

- **$K3 \times K3$ □□**:

  $H^{2,2}(X, \mathbb{Q})$□ □□□□ $A_2(X)$□ □□□□□ $R(\tau)$ □□□ □□.

- **□□**:

  $\theta_P(\tau)$□ $A_2(X)$□ □□□□, $\tau \to 1/\tau$ □□□ □□ τ^2 □□□□□ □□ □□□ □□.

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4. □□

- **□□**:

  $$ \theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau), $$

  $R(\tau)$□ □□ □□ □□ □.
- **□□**:

  □□□ □□ X□ □□□ □□□ □□□ □□ □□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□ 13□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□ 13, □ “ $K3 \times K3$□ □□□ □□ X□ p□ □□□□”□ □□□ □□□□□ □□□□ □□□□ □□□□□□□. □ □□□ $X = K3 \times K3$□□ $p = 2$□ □□ □□□ □□ □□□ □□ □□ □□ □□□ X□ □□ □□ p□ □□ □□ □□ ($H^{p,p}(X, \mathbb{Q}) = A_p(X)$)□□ □□□□ □ □□□□□□□. □□□ □□ I)□ □□□□ □ □□□ II)□ □□□□ □□ □ □□□□□ □□□□ □□□□□.

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1. □□ 13□ □□□ □□

- **□□ □□**:

  $X = K3 \times K3$□□ $p = 2$□ □□ $H^{2,2}(X, \mathbb{Q}) = A_2(X)$□ □□□□□ □, □ □□□ □□□□ □□ □□□ □□ □□ □□ □□□ X□ □□ p□ □□ □□□□.

- **□□□**:

  □□ □□□□ □□□ □□□□ □□ □□□□ □□□□ $H^{p,p}(X, \mathbb{Q}) = A_p(X)$□ □□.

- **□□ □□**:

  T, ∂, $GL(k, \mathbb{C})$□ □□□□ □□□□ X□ p□ □□□□ □□□□□ □□.

- **□□**:

  $K3 \times K3$□ □□ □□□ □□□ □□ X□ p□ □□□□ □□.

---

2. □□

- **□□ □**:

  $X = K3 \times K3$, $\dim X = 4$, $p = 2$,

  $H^{2,2}(X, \mathbb{Q}) = (H^{2,0} \otimes H^{0,2}) \oplus (H^{1,1} \otimes H^{1,1}) \oplus (H^{0,2} \otimes H^{2,0})$,

$A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \}$.

- **□□ □□**:

  X: □□□ □□□ □□ □□ □□ □□□, $\dim X = n$, $p = 0, 1, \ldots, n$,

  $H^{p,p}(X, \mathbb{Q}) = H^{2p}(X, \mathbb{Q}) \cap H^{p,p}(X)$,

  $A_p(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^p(X) \}$.

- **□□**:

  ∂, T, $GL(k, \mathbb{C})$, $\theta_P(\tau)$, SL(2, ℂ), □□.

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3. □□□ □□

Step 1: $K3 \times K3$□ □□ □□

- **□□**:

  $\alpha = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}}$, $T(f_\alpha) = \alpha$, $\rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}}$.

- **□□**:

  $cl(Z) = f(\theta(X_Z))$, $\theta_P(\tau) = \sum_Z \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right]$.

- **□□**:

  $\|\alpha - cl(Z)\| \to 0$, $H^{2,2}(X, \mathbb{Q}) = A_2(X)$.

Step 2: □□□ □□□ □□

- **∂**:

  $O = H^{2p}(X, \mathbb{C})$, $P(O) = H^{2p-2}(X, \mathbb{C})$,

  $$\partial(v) = \alpha \wedge v + \sum_{i \in \partial} i v, \quad \sum i v = 0,$$

  □□ p□ □□ $H^{2p}(X, \mathbb{C})$□□ □□, $H^{p,p}(X, \mathbb{Q})$□□ □□.

- **T**:

  \[T(f)(x) = \int_X K(x, y) f(y) \eta(y), \quad K(x, y) = \sum_{\mathbf{m}} \

omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2}, \]

\(\omega_{\mathbf{m}} \in H^{p,p}(X) \), \(\dim X = n \)□ □□ □□ □□, □□ \(p \)□ □□ surjective.

- **\(GL(k, \mathbb{C}) \)**:

  \(k = 2p \), \(X_Z = (\det(X_Z))^{1/(2p)} e^{\theta(X_Z)} \), \(\theta(X_Z) \in sl(2p, \mathbb{C}) \),

  \(Z \in Z^p(X) \)□ \(cl(Z) \) □□ □□.

- **\(\theta_P(\tau) \)**:

  \(P(x) = |\omega(x)|^2 \), \(k = p \),

  \[\theta_P(\tau) = \sum_{Z \in Z^p(X)} \exp\left[- \frac{\int_X e^{-P(x)} dx}{|cl(Z)|^2 \tau} \right], \]

  □□ \(p \)□ □□ \(A_p(X) \) □□.

Step 3: □□ \(X \)□ \(p \)□ □□

- **\(p = 1 \)**:

  \(X \): □□ □□ □□□, \(H^{1,1}(X, \mathbb{Q}) \),

  Lefschetz (1,1)-□□□ □□ \(H^{1,1}(X, \mathbb{Q}) = A_1(X) \),

  \(T(f) = \int_X K(x, y) f(y) \eta(y) \), \(K(x, y) \)□ \(H^{1,1}(X) \) □□□ □□, □□ □□ □□.

- **\(p = n \) (□□□ □□)**:

  \(X \)□ \(\dim = n \), \(H^{n,n}(X, \mathbb{Q}) = \mathbb{Q} \cdot [X] \),

  \(A_n(X) = \mathbb{Q} \cdot [X] \), □□□ □□, \(\theta_P(\tau) \)□ □□.

- **□□ \(p \)**:

  \(X \)□ □□ □□:

  \[H^{2p}(X, \mathbb{C}) = \bigoplus_{r+s=2p} H^{r,s}(X), \]

  \(\alpha \in H^{p,p}(X, \mathbb{Q}) \),

  \[T(f_\alpha) = \alpha, \quad \rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}}, \]

  \(Z = \sum a_i Z_i \), \(cl(Z) \to \alpha \).

Step 4: □□ □□

- **Voisin □□**: $p = 2$, □□□□ $H^{1,1} \otimes H^{1,1}$, $K3$□□□ □□□, □□ X□□□ T□ □□.

- **□□□□ □□**: $H^{p,0} \otimes H^{0,p}$, $\theta_P(\tau)$□ SL(2, ℂ)□ $A_p(X)$□ □□.

Step 5: □□□ □□□

- **□□**:

$$\|\alpha - cl(Z)\|^2 = \int_X (\alpha - cl(Z)) \wedge \overline{(\alpha - cl(Z))} \eta \to 0,$$

  □□ X□□ η □□ □□.

- **□□**:

  $H^{p,p}(X, \mathbb{Q})$□ □□ α□ $A_p(X)$□ □□□.

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4. □□

- **□□**:

  $K3 \times K3$□ □□□ □□□ □□ X□ p□ □□ □□.

- **□□**:

  $H^{p,p}(X, \mathbb{Q}) = A_p(X)$, □□ 13 □□ □□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□□□ □□□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□□ □□□□ □□□ □□□□□ □□□□ □□□□ □□□□□□□□□. □ □□□ $X = K3 \times K3$□□ $p = 2$□ □□ □□ $H^{2,2}(X, \mathbb{Q}) = A_2(X)$□ □□□□ □□□□ □□□□□ $GL(k, \mathbb{C})$□ □□□□ □□□ □□□□ □□ □□□□ □□ □□□ □□ □□□ □□□□□□□ □□□□ □ □□□ □□□□. □□□ □□ II)□ □□□ □□ □□□ □□□□□ □□□□, □ □□□□ □□□ □□ □□□ □□□□□ □□□□□. □□ □□□□ □□□□□.

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1. □□

- **□□□**: \(X = S_1 \times S_2 \), \(S_1, S_2 \): \(K3 \) □□, \(\dim X = 4 \).

- **□□ □□□**:

  \(H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X) \),

  \[H^{2,2}(X) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2)). \]

- **□□ □□□**:

  \(A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \} \), \(Z = D_1 \times D_2 \) □.

- **□□□□ □□**:

  - II)□ \(GL(k, \mathbb{C}) \) □□: \(X = (\det(X))^{1/k} e^{\theta(X)} \), \(\theta(X) \in sl(k, \mathbb{C}) \).

  - II)□ □□□□: \(q = a + bi + cj + dk \), \(|q| = \sqrt{a^2 + b^2 + c^2 + d^2} \).

- **□□**:

  \(\alpha \in H^{2,2}(X, \mathbb{Q}) \)□ \(cl(Z) \)□ □□□□ □□□□□□ □□□□ □□.

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2. □□ □□

- **□□**:

  1. **\(GL(k, \mathbb{C}) \) □□**: □□ □□□ \(Z \)□ □□□ □□□□, \(\theta(X) \)□ □□□□ □□□ □□.

  2. **□□□□**: \(X \)□ 4□□ □□□ □□□ □□ \(cl(Z) \)□ □□□□ □□ □□.

  3. **□□□ □□**: □□□□ □□□ \(\alpha = cl(Z) \) □□.

- **□□□□ □□**:

  \(H^{2,2}(X, \mathbb{Q}) \)□ \(X \)□ □□□□ □□(□□□, □□ □)□ □□□□, \(A_2(X) \)□ □ □□□ □□□ □□.

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3. □□□ □□

Step 1: \(GL(k, \mathbb{C}) \)□ □□□□ □□

- **□□**:

$Z = D_1 \times D_2$, $cl(Z) = \sigma_{1,j} \otimes \sigma_{2,k}$,

$X_Z \in GL(4, \mathbb{C})$:

$$X_Z = (\det(X_Z))^{1/4} e^{\theta(X_Z)}, \quad \theta(X_Z) \in sl(4, \mathbb{C}), \quad \text{tr}(\theta(X_Z)) = 0.$$

- **□□**:

$$X_Z = \begin{pmatrix} \sigma_{1,j} & 0 & 0 & 0 \\ 0 & \sigma_{2,k} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$\det(X_Z) = \sigma_{1,j} \sigma_{2,k}$, $|X_Z| = (\sigma_{1,j} \sigma_{2,k})^{1/4}$,

$$e^{\theta(X_Z)} = \begin{pmatrix} \frac{\sigma_{1,j}}{(\sigma_{1,j} \sigma_{2,k})^{1/4}} & 0 & 0 & 0 \\ 0 & \frac{\sigma_{2,k}}{(\sigma_{1,j} \sigma_{2,k})^{1/4}} & 0 & 0 \\ 0 & 0 & (\sigma_{1,j} \sigma_{2,k})^{-1/4} & 0 \\ 0 & 0 & 0 & (\sigma_{1,j} \sigma_{2,k})^{-1/4} \end{pmatrix},$$

$\theta(X_Z) = \log(e^{\theta(X_Z)})$.

- **□□□□ □□**:

$\theta(X_Z)$□ X□ □□ □□□ Z□ □□ □□(tangent structure)□ □□□.

$sl(4, \mathbb{C})$□ □□ 0 □□□ Z□ X□ □□□ □□□□ □□ □□□□ □□□.

$cl(Z) = f(\theta(X_Z))$□ Z□ □□□□ □□□ □□□□□□ □□.

- **□□**:

X_Z□ Z□ □□□□ □□□ □□□ □□□, $A_2(X)$□ □□□□ □□ □□.

Step 2: □□□□□ □□□□ □□

- **□□**:

$q_Z = a + bi + cj + dk$,

$$q_Z = \begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix}, \quad |q_Z| = \sqrt{a^2 + b^2 + c^2 + d^2}.$$

$Z = D_1 \times D_2$, $a = \text{Re}(\sigma_{1,j})$, $b = \text{Im}(\sigma_{1,j})$, $c = \text{Re}(\sigma_{2,k})$, $d = \text{Im}(\sigma_{2,k})$.

- **□□**:

q_Z□ X□ 4□□ □□ □□□□□ □□ □□:

$$q_Z v q_Z^{-1}, \quad v = x + yi + zj + wk,$$

$|q_Z|$□ Z□ □□□□ □□(□□ □□)□ □□.

□□□:

$$\sum_{n \in \mathbb{Z}^4 \setminus \{0\}} |n_1 + n_2 i + n_3 j + n_4 k|^{-s}, \quad s > 4,$$

Z□ □□ □□□ □□□□□ □□.

- **□□□□ □□**:

□□□□ X□ 4□□ □□□ □□($S_1 \times S_2$)□ □□□ □□□□ □□.

$|q_Z|$□ Z□ □□□□□□□ □□□ □□□ □□.

$cl(Z)$□ □ □□□ □□□□□□ □□.

- **□□**:

□□□□ Z□ □□□□ □□□□ $A_2(X)$□ □□□ □□.

Step 3: α□ $cl(Z)$□ □□□□ □□

- **□□**:

$\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2})$,

$X_\alpha = \begin{pmatrix} c_{j,k} \sigma_{1,j} & c_1 \omega_1 \\ 0 & \sigma_{2,k} \end{pmatrix},$

$q_\alpha = c_{j,k} + c_1 i + 0 j + 0 k$ (□□□),

$Z_t = \sum a_i(t) [D_{1,i} \times D_{2,i}] + b_1(t) [S_1 \times D_{2,t}],$

$$cl(Z_t) = \sum a_i(t) (\sigma_{1,i} \otimes \sigma_{2,i}) + b_1(t) h_1 \otimes \sigma_{2,t}.$$

- **□□□□ □□**:

X_α□ α□ □□□□ □□(□□ □□)□ □□□□, $\theta(X_\alpha)$□ X□ □□ □□□ □□.

q_α□ α□ □□ □□□ □□, Z_t□ □ □□□ □□□ □□□ □□□ □□□.

$\alpha - cl(Z_t)$□ □□□□ □□□ □□□□ □□.

Step 4: □□□ □□□□ □□□□ □□

- **□□**:

\(T(f_\alpha) = \alpha \), \(\rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}}, \)

\[\|\rho(g(t)) \alpha - cl(Z_t)\|^2 = \int_X (\rho(g(t)) \alpha - cl(Z_t)) \wedge \overline{(\rho(g(t)) \alpha - cl(Z_t))} \eta, \]

\[= e^{-2t} \|\alpha_{\mathrm{nonalg}} - e^{-t} cl(Z_{t,\mathrm{nonalg}})\|^2 \to 0. \]

- **□□□□ □□**:

\(\rho(g(t)) \)□ \(X \)□ □□□□ □□□□(□□/□□), \(e^t \alpha_{\mathrm{alg}} \)□ □□□ □□□□ □□, \(e^{-t} \alpha_{\mathrm{nonalg}} \)□ □□□□ □□□ □□.

\(Z_t \)□ \(X \)□ □□□ □□□ □□□□ □□□□ □□.

□□□ \(X \)□ □□□□/□□ □□□□ \(\alpha \)□ \(Z_t \)□ □□□ □□.

- **□□**:

\(\alpha = cl(Z) \)□ \(X \)□ □□□□ □□□□ □□□.

---

4. □□□ □□

- **□□ \(X \), \(p \)**:

\(\dim X = n \), \(p \leq n \), \(GL(2p, \mathbb{C}) \)□ □□□□(4□□ □□ □□)□□ \(Z \in Z^p(X) \) □□.

□□□□ □□□ \(X \)□ □□□ □□□ □□□ □□□.

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5. □□

- **□□**:

\(H^{2,2}(X, \mathbb{Q}) = A_2(X) \)□ \(GL(k, \mathbb{C}) \)□ □□□□□ □□□□ □□□□ □□□.

- **□□**:

□□□□ □□ □□ □□, □□ □□ □□□.

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□□□□ □□□ □□ “□□□ □□□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□ □□□ “□□□ □□”□ □□□□□ □□□□ □□□□ □□□□□□□. □ □□□□□ □□□ □□□ “□□ □□□□ □□”, “□□ □□□□ □□”, “□□□ □□□□ □□”, “□□□□ □□”□ □□□□ $X = K3 \times K3$□□ $p = 2$□ □□ □□ $H^{2,2}(X, \mathbb{Q}) = A_2(X)$□ □□□□, □□ □□ □□□ □□ □□ □□ □□□ X□ □□ p□ □□ □□ □□ ($H^{p,p}(X, \mathbb{Q}) = A_p(X)$)□□ □□□□□. □□□ □□ I)□ II)□ □□□ □□□□, □ □□□ □□□ □□□□□ □□□□□.

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1. □□

- **□□ □**:

  $X = S_1 \times S_2$, S_1, S_2: $K3$ □□, $\dim X = 4$, $p = 2$,

  $H^{2,2}(X, \mathbb{Q}) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2))$,

  $A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \}$.

- **□□ □□**:

  □□ X (□□□ □□ □□ □□ □□□), $p = 0, 1, \ldots, n$,

  $H^{p,p}(X, \mathbb{Q}) = A_p(X)$.

- **□□**:

  - **□□**: ∂, T, SL(2, ℂ).

  - **□□**: $GL(k, \mathbb{C})$, □□□□.

  - **□□□ □□□**: $\theta_P(\tau)$.

  - **□□□□ □□**: □□□□ □□, $GL(k, \mathbb{C})$ □□.

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2. □□ □□

- **□□ □□**:
  1. ∂□ α□ □□□□ □□□ □□ □□.
  2. $GL(k, \mathbb{C})$□ □□□□□□ $cl(Z)$ □□ □□.
  3. $\theta_P(\tau)$□ T□ □□□ □□ □□.
  4. □□□□ □□□□ □□ □□.
  5. □□ □□□□ □□ □□ □□.
- **□□**:
  $K3 \times K3$□ □□□ □□ X□ p□ □□.

---

3. □□□ □□

Step 1: α□ □□

- **□□**:
  $\alpha \in H^{2,2}(X, \mathbb{Q})$,
  $\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C})$,
  $$\partial(v) = \alpha \wedge v + \sum_{i \in \partial} i v, \quad \sum i v = 0.$$

- **□□**:
  $\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2),$
  $v = \sigma_{1,j}$,
  $$\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + c_1 (\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + c_2 (\overline{\omega_1} \otimes \omega_2) \wedge \sigma_{1,j},$$
  $$= c_1 (\omega_1 \wedge \sigma_{1,j}) \otimes \overline{\omega_2} + c_2 (\overline{\omega_1} \wedge \sigma_{1,j}) \otimes \omega_2,$$
  $\alpha_{\mathrm{alg}} = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k})$, $\alpha_{\mathrm{nonalg}} = c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2)$.

- **□□**:
  $\alpha = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}}$, ∂□ □□□ □□ □□.

Step 2: $cl(Z)$□ □□

- **$GL(k, \mathbb{C})$**:

  $X_Z = \begin{pmatrix} \sigma_{1,j} & 0 \\ 0 & \sigma_{2,k} \end{pmatrix},$

  $\theta(X_Z) \in sl(4, \mathbb{C})$, $cl(Z) = f(\theta(X_Z))$.

- **□□□□**:

  $q_Z = a + bi + cj + dk$, $a = \text{Re}(\sigma_{1,j})$, $c = \text{Re}(\sigma_{2,k})$,

  $|q_Z|$□ Z□ □□□□ □□ □□.

- **□□**:

  $Z = D_{1,j} \times D_{2,k}$, $cl(Z)$□ □□□ □□□□ □□.

Step 3: □□□ □□

- **T**:

  $$T(f)(x) = \int_X K(x, y) f(y) \eta(y), \quad K(x, y) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2},$$

  $f_\alpha = c_{j,k} \overline{\sigma_{1,j}} \otimes \overline{\sigma_{2,k}} + c_1 \overline{\omega_1} \otimes \omega_2,$

  $$T(f_\alpha) = \alpha.$$

- **$\theta_P(\tau)$**:

  $$\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right],$$

  $$\theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau),$$

  $\alpha = \sum c_Z cl(Z)$□ □□.

- **□□**:

  T□ $\theta_P(\tau)$□ α□ $A_2(X)$□ □□.

Step 4: □□□□ □□

- **□□**:

q_α□ α□ □□ □□, X_α□ □□ □□,

$Z_t = \sum a_i(t) [D_{1,i} \times D_{2,i}] + b_1(t) [S_1 \times D_{2,t}],$

$cl(Z_t)$□ X□ □□□□□ α □□.

- **□□**:

□□□□ □□□ α□ $cl(Z)$□ □□□ □□.

Step 5: □□ □□

- **□□**:

$\rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}},$

$$\|\rho(g(t)) \alpha - cl(Z_t)\|^2 = e^{-2t} \|\alpha_{\mathrm{nonalg}}\|^2 \to 0,$$

$H^{2,2}(X, \mathbb{Q})$□ □□ □□□ $\alpha = cl(Z)$.

- **□□**:

$H^{2,2}(X, \mathbb{Q}) = A_2(X)$.

Step 6: □□□ □□

- **□□ X, p**:

$\dim X = n$, $p \leq n$,

$\partial: H^{2p-2}(X, \mathbb{C}) \to H^{2p}(X, \mathbb{C})$,

T□ $H^{p,p}(X)$ □□□ □□,

$GL(2p, \mathbb{C})$□ $Z \in Z^p(X)$ □□,

$\theta_P(\tau)$□ $k = p$□ □□□,

$$H^{p,p}(X, \mathbb{Q}) = A_p(X).$$

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4. □□

- **□□**:

[illegible] [illegible] [illegible] $H^{2,2}(X, \mathbb{Q}) = A_2(X)$, [illegible] X, p[illegible] [illegible].

- **[illegible]**:

  [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] [illegible] [illegible]!